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Multiple Attribute Group Decision Making for Plant Location Selection with Pythagorean Fuzzy Weighted Geometric Aggregation Operator

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ABSTRACT

There are many aggregation operators and applications have been developed up to date, but in this paper we present the idea of Pythagorean fuzzy weighted geometric aggregation operator, and also discuss some of their basic properties. At the last we give an application of this proposed operator. For this purpose we construct an algorithm and also construct a numerical example.

1. Introduction

The idea of fuzzy set was familiarized by L. A. Zadeh in 1965 [1].In 1986, Atanassov presented the idea of IFS, which a general form of the FS [2]. The intuitionistic fuzzy set has gotten increasingly consideration since its development [3, 4, 5, 6, 7, 8, 9, 10, 11]. Bostince and Burillo [12] demonstrated that vague sets are mathematically equal to IFS. De at al [13]demarcated dilation normalization and concentration, of IFS. He additionally demonstrated several recommendations in the proposed field. Bostince et al. [14] introduced the notion of intuitionistic fuzzy generators and also deliberate the corresponding of IFS from the intuitionistic fuzzy generators. Yager [15, 16] introduced the notion of PFS. Xu [17] established several operators such as, (IFWA, IFOWA, IFHA) operators. After the introduction of arithmetic aggregation operator, Xu and Yager [18] industrialized geometric aggregation operators, such as (IFWG, IFOWG, IFHG) operators. They also applied them to MAGDM based on IFSs. Wei [19] introduced the notion of the induced geometric aggregation operators with IFI and they also using these operators for group decision making. Liu [20] introduced the notion of (IFEWG, IFEOWG) operators. Bellman and L. A. Zadeh [21] presented the theory of fuzzy sets in the MAGDM problems. IFSs have got great focus [22-24]. In 2015, X. Peng and Y. Yang [25] introduced the notion of PFWA operator, PFWPA operator , PFWPG operators. In [26, 27] Xu and R. R. Yager also worked in the field if intuitionistic aggregation operators.

Thus keeping the advantage of the above aggregation operators in this paper we introduce the notion of Pythagorean fuzzy weighted geometric aggregation operator and also discuss some of their properties.

This paper consists of six section. In section 2, we give some main definitions and results which can be used in our late discussion. In section 3, we explain some new operational laws and relations on PFS. In section 4, we develop PFWG operator and also explain some of their properties. Section 5 containing an algorithm for MAGDM. In part 6, we have.

1. Preliminaries

***Definition 2.1 [13]*:** Let Z is a fixed set, and then a fuzzy set can be defined as:

$$V=\left\{\left(z,\mu_V(z)\right)\mid z\in Z\right\} \tag{1}$$

where μ_V is a mapping from Z to $[0,1]$, and $\mu_V(z)$ is said to be the degree of membership of element *z* in *Z.*

***Definition 2.2 [5]*:** Let Z is a fixed set, then an intuitionistic fuzzy set can bedefined as:

$$L=\left\{\left(z,\mu_L(z),\eta_L(z)\right)\mid z\in Z\right\} \tag{2}$$

where $\mu_L(z)$ and $\eta_L(z)$ are mappings from Z to [0.1], with some conditions such that

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$0 \le \mu_L(z) \le 1,\ 0 \le \eta_L(z) \le 1$ and $0 \le \mu_L(z) + \eta_L(z) \le 1, \forall z \in Z$.

Definition 2.3: [17] Let K be a universal set, then a Pythagorean fuzzy set, P in K can be defined as:

$$P = \{\langle k, u_P(k), v_P(k)\rangle \mid k \in K\}, \tag{3}$$

where $u_P(k) : P \to [0,1], v_P(k) : K \to [0,1]$ are called membership and non-membership functions of $k \in K$ respectively, with condition $0 \le (u_P(k))^2 + (v_P(k))^2 \le 1$, for all $k \in K$. Let $\pi_P(k) = \sqrt{1 - u_P^2(k) - v_P^2(k)}$, then it is called the Pythagorean fuzzy index of $k \in K$ with condition $0 \le \pi_P(k) \le 1$, for every $k \in K$.

Definition 2.4 [22]: Let $\rho = (\mu_\rho, \eta_\rho)$, $\rho_1 = (\mu_{\rho_1}, \eta_{\rho_1})$, $\rho_2 = (\mu_{\rho_2}, \eta_{\rho_2})$, are three PFNs *and* $\Upsilon > 0$. *Then*

(1) $\rho^c = (\eta_\rho, \mu_\rho)$,

(2) $\rho_1 \oplus \rho_2 = \left(\sqrt{\mu_{\rho_1}^2 + \mu_{\rho_2}^2 - \mu_{\rho_1}^2 \mu_{\rho_2}^2}, \eta_{\rho_1}\eta_{\rho_2}\right)$,

(3) $\rho_1 \otimes \rho_2 = \left(\mu_{\rho_1}\mu_{\rho_2}, \sqrt{\eta_{\rho_1}^2 + \eta_{\rho_2}^2 - \eta_{\rho_1}^2 \eta_{\rho_2}^2}\right)$,

(4) $\Upsilon\rho = \left(\sqrt{1 - \left(1 - \mu_\rho^2\right)^\Upsilon}, \eta_\rho^\Upsilon\right)$,

(5) $\rho^\Upsilon = \left(\mu_\rho^\Upsilon, \sqrt{1 - \left(1 - \eta_\rho^2\right)^\Upsilon}\right)$.

Definition 2.5 [22]: Let $\rho = (\mu_\rho, \eta_\rho)$ be a PFV, then we can find the score of ρ as following:

$$S(\rho) = \mu_\rho^2 - \eta_\rho^2, \tag{4}$$

where $S(\rho) \in [-1,1]$.

Definition 2.6 [22] : Let $\rho = (\mu_\rho, \eta_\rho)$ be a PFN, then the accuracy degree ρ can be defined as follows:

$$H(\rho) = \mu_\rho^2 + \eta_\rho^2, \tag{5}$$

where $H(\rho) \in [0,1]$.

Definition 2.7: Let $\rho = (0.8, 0.6)$, then

$S(\rho) = (0.8)^2 - (0.6)^2 = 0.28$ *and*

$H(\rho) = (0.8)^2 + (0.6)^2 = 1$

Definition 2.8 [22]: Let $\rho_1 = (\mu_{\rho_1}, \eta_{\rho_1})$ and $\rho_2 = (\mu_{\rho_2}, \eta_{\rho_2})$ be the two Pythagorean fuzzy numbers, then $S(\rho_1) = \mu_{\rho_1}^2 - \eta_{\rho_1}^2$, $S(\rho_2) = \mu_{\rho_2}^2 - \eta_{\rho_2}^2$, $H(\rho_1) = \mu_{\rho_1}^2 + \eta_{\rho_1}^2$, $H(\rho_2) = \mu_{\rho_2}^2 + \eta_{\rho_2}^2$ are the scores and accuracy of ρ_1 and ρ_2 respectively. Then the following holds:

(1) If $S(\rho_2) \succ S(\rho_1)$, then ρ_2 is greater than ρ_1 represented by $\rho_1 < \rho_2$,

(2) If $S(\rho_1) = S(\rho_2)$, then

(a) If $H(\rho_1) = H(\rho_2)$, then, ρ_1 and ρ_2 have the same information i.e., $\mu_{\rho_1} = \mu_{\rho_2}$ and $\eta_{\rho_1} = \eta_{\rho_2}$ represented by $\rho_1 = \rho_2$.

(b) If $H(\rho_1) < H(\rho_2)$ then ρ_2 is greater than ρ_1

Definition 2.9 [24]: Let $\rho_j = (\mu_{\rho_j}, \eta_{\rho_j})(j = 1,2,...,n)$ be a collection of *IFVs* and let $IFWG : \Omega^n \to \Omega$, then the intuitionistic fuzzy set can be define as following:

$$IFWG_\omega(\rho_1, \rho_2, ..., \rho_n) = \rho_1^{\omega_1} \otimes \rho_2^{\omega_2} \otimes ... \oplus \rho_n^{\omega n} \tag{6}$$

where $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ is the weighted vector of $\rho_j (j = 1,2,...,n)$ such that, $\omega_j \in [0,1]$ and also $\sum_{j=1}^n \omega_j = 1$. Mostly, if $\omega = \left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)^T$, then *IFWG* operator is reduced to an *IFG* operator of dimension *n*, which can be demarcated as ensuing:

$$IFG(\rho_1, \rho_2, ..., \rho_n) = (\rho_1 \otimes \rho_2 \otimes ... \otimes \rho_n)^{\frac{1}{n}} \tag{7}$$

Definition 2.10 [24]: Let $\rho_j = (\mu_{\rho_j}, \eta_{\rho_j})(j = 1,2,...,n)$ be the assortment of *IFVs*. Then *IFOWG* operator of dimension *n* is a mapping $IFOWG : \Omega^n \to \Omega$, and also that has an associated vector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$, with some conditions $\omega_j \in [0,1]$ and $\sum_{j=1}^n \omega_j = 1$. Also

$$IFOWG_\omega(\rho_1, \rho_2, ..., \rho_n) = \left(\rho_{\sigma(1)}\right)^{\omega_1} \otimes \left(\rho_{\sigma(2)}\right)^{\omega_2} \otimes ... \otimes \left(\rho_{\sigma(n)}\right)^{\omega_n} \tag{8}$$

We also know that $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of (1,2, …, n) such that $\rho_{\sigma(j-1)} \ge \rho_{\sigma(j)}$ for all j. Particularly, if $\omega=\left(\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right)^T$, then IFOWG operator is reduced to *IFG* operator of n dimension.

2. Operational Lawsand Relations

Theorem 3.1: Let $\rho_1=\left(\mu_{\rho_1},\eta_{\rho_1}\right)$ and ρ_2 $p_2=\left(\mu_{\rho_2},\eta_{\rho_2}\right)$ be the two *PFVs*, and let $\Upsilon_1=\rho_1\otimes\rho_2$ and $\Upsilon_2=\rho^{\Upsilon}\left(\Upsilon>0\right)$. Then Υ_1 and Υ_2 are also *PFVs*.

Proof **:** Since $\rho_1=\left(\mu_{\rho_1},\eta_{\rho_1}\right)$ and $p_2=\left(\mu_{\rho_2},\eta_{\rho_2}\right)$ are the two PFVs , then we have

$$\mu_{\rho_1}\in(0,1),\eta_{\rho_1}\in(0,1),\mu_{\rho_2}\in(0,1),\eta_{\rho_2}\in(0,1)$$

and $\mu_{\rho_1}^2+\eta_{\rho_1}^2\le 1,\ \mu_{\rho_2}^2+\eta_{\rho_2}^2\le 1.$ Hence

$$\begin{aligned}&\left(\mu_{\rho_1}\mu_{\rho_2}\right)^2+\left(\sqrt{\eta_{\rho_1}^2+\eta_{\rho_2}^2-\eta_{\rho_1}^2\eta_{\rho_1}^2}\right)^2\\ &\le\left(1-\eta_{\rho_1}^2\right)\left(1-\eta_{\rho_2}^2\right)+\left(\sqrt{\eta_{\rho_1}^2+\eta_{\rho_2}^2-\eta_{\rho_1}^2\eta_{\rho_1}^2}\right)^2\\ &=\left(1-\eta_{\rho_1}^2\right)\left(1-\eta_{\rho_2}^2\right)+\eta_{\rho_1}^2+\eta_{\rho_2}^2-\eta_{\rho_1}^2\eta_{\rho_1}^2\\ &=1.\end{aligned}$$

Thus Υ_1 is a *PFV* . Now let $\mu_{\rho}^{\Upsilon}\ge 0$ and $\eta_{\rho}^{\Upsilon}\ge 0.$ Since

$$\begin{aligned}&\left(\mu_{\rho}^{\Upsilon}\right)^2+\left(\sqrt{1-\left(1-\eta_{\rho}^2\right)^{\Upsilon}}\right)^2\\ &\le\left(1-\eta_{\rho}^2\right)^{\Upsilon}+\left(\sqrt{1-\left(1-\eta_{\rho}^2\right)^{\Upsilon}}\right)^2\\ &=\left(1-\eta_{\rho}^2\right)^{\Upsilon}+1-\left(1-\eta_{\rho}^2\right)^{\Upsilon}\\ &=1.\end{aligned}$$

Thus Υ_2 is also a PFV.

There are some special cases, now we are going to discuss these cases in detail in the following.

(1) If $p=\left(\mu_{\rho},\eta_{\rho}\right)=(1,1)$ i.e. $\mu_{\rho}=1,\eta_{\rho}=1,$

then $\rho^{\Upsilon}=(1,1).$

$$\begin{aligned}\rho^{\Upsilon}&=\left(\mu_{\rho}^{\Upsilon}\sqrt{1-\left(1-\eta_{\rho}^2\right)^{\Upsilon}}\right)=\left(1,\sqrt{1-(1-1)^{\Upsilon}}\right)\\ &=\left(1,\sqrt{1-(0)}\right)=\left(1,\sqrt{1}\right)=(1,1).\end{aligned}$$

(2) If $p=\left(\mu_{\rho},\eta_{\rho}\right)=(0,0),$ i.e, $\mu_{\rho}=0,\ \eta_{\rho}=0,$

then $\rho^{\Upsilon}=(0,0)$

$$\begin{aligned}\rho^{\Upsilon}&=\left(\mu_{\rho}^{\Upsilon},\sqrt{1-\left(1-\eta_{\rho}^2\right)^{\Upsilon}}\right)=\left(0,\sqrt{1-(1-0)^{\Upsilon}}\right)\\ &=\left(0,\sqrt{1-(1)^{\Upsilon}}\right)=\left(0,\sqrt{1-1}\right)=(0,0).\end{aligned}$$

(3) If $p=\left(\mu_{\rho},\eta_{\rho}\right)=(0,1)$ i.e., $\mu_{\rho}=0,\ \eta_{\rho}=1,$

then $\rho^{\Upsilon}=(0,1)$

$$\begin{aligned}\rho^{\Upsilon}&=\left(\mu_{\rho}^{\Upsilon},\sqrt{1-\left(1-\eta_{\rho}^2\right)^{\Upsilon}}\right)=\left(0,\sqrt{1-(1-1)^{\Upsilon}}\right)\\ &=\left(0,\sqrt{1-(0)}\right)=(0,1).\end{aligned}$$

(4) If $\Upsilon\to 0$ and $0\le\mu_{\rho},\eta_{\rho}\le 1,$ then

$\rho^{\Upsilon}=\left(\mu_{\rho},\eta_{\rho}\right)\to(1,0)$ i.e. $\rho^{\Upsilon}\to(1,0)\left(\Upsilon\to 0\right)$

$$\begin{aligned}\rho^{\Upsilon}&=\left(\mu_{\rho}^{\Upsilon},\sqrt{1-\left(1-\eta_{\rho}^2\right)^{\Upsilon}}\right)=\left(1,\sqrt{1-(1-1)^{\Upsilon}}\right)\\ &=\left(1,\sqrt{1-(1-1)^{0}}\right)=\left(1,\sqrt{1-1}\right)=(1,0).\end{aligned}$$

(5) If $\Upsilon\to+\infty$ and $0\le\mu_{\rho},\eta_{\rho}\le 1,$ then

$\rho^{\Upsilon}=\left(\mu_{\rho},\eta_{\rho}\right)\to(0,1)$ i. e. $\rho^{\Upsilon}\to(0,1)\left(\Upsilon\to+\infty\right)$

$$\begin{aligned}\rho^{\Upsilon}&=\left(\mu_{\rho}^{\Upsilon},\sqrt{1-\left(1-\eta_{\rho}^2\right)^{\Upsilon}}\right)=\left(0,\sqrt{1-(1-1)^{\Upsilon}}\right)\\ &=\left(0,\sqrt{1-(0)^{\Upsilon}}\right)=\left(0,\sqrt{1}\right)=(0,1).\end{aligned}$$

$\Upsilon=1,$ then $\rho^{\Upsilon}=\left(\mu_{\rho},\eta_{\rho}\right).$ i.e.

$\rho^{\Upsilon}\to\rho\left(\Upsilon=1\right)$

$$\begin{aligned}\rho^{\Upsilon}&=\left(\mu_{\rho}^{\Upsilon},\sqrt{1-\left(1-\eta_{\rho}^2\right)^{\Upsilon}}\right)=\left(\mu^{1}{}_{\rho},\sqrt{1-\left(1-\eta_{\rho}^2\right)^{1}}\right)\\ &=\left(\mu_{\rho},\sqrt{1-\left(1-\eta_{\rho}^2\right)}\right)=\rho.\end{aligned}$$

Definition 4.1: Let $\rho_j=\left(\mu_{\rho_j},\eta_{\rho_j}\right)\left(j=1,...,n\right)$ be PFVs and let *PFWG* : $\Omega^n\to\Omega,$ then the Pythagorean fuzzy weighted geometric aggregation operator can be define as:

$$PFWG_{\omega}\left(\rho_1,\rho_2,...,\rho_n\right)=\rho_1^{\omega_1}\otimes\rho_2^{\omega_2}\otimes...\oplus\rho_n^{\omega_n}\quad(9)$$

Where $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ is the weighted vector of $\rho_j\,(j=1,2,3,...,n)$ with condition $\omega_j\in[0,1]$ and $\sum_{j=1}^{n}\omega_j=1$. If $\omega=\left(\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right)^T$, then the PFWG operator is converted to *PFG* operator which is defined as:

$$PFG(p_1,p_2,...,p_n)=(\rho_1\otimes\rho_2\otimes...\otimes\rho_n)^{\frac{1}{n}} \qquad (10)$$

Example 4.2: Let

$$\rho_1=(0.4,0.6),\rho_2=(0.5,0.7),$$
$$\rho_3=(0.3,0.8),\rho_4=(0.2,0.9)$$

Thus

$$PFWG_w(\rho_1,\rho_2,\rho_3,\rho_4)$$
$$=\left(\prod_{j=1}^{4}\mu_{\rho_j}^{\omega_j},\sqrt{1-\prod_{j=1}^{4}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}\right)$$
$$=\begin{pmatrix}(0.4)^{0.1}\times(0.5)^{0.2}\times(0.3)^{0.3}\times(0.2)^{0.4},\\ \sqrt{\begin{array}{c}1-(1-0.36)^{0.1}(1-0.49)^{0.2}\\(1-0.64)^{0.3}(1-0.81)^{0.4}\end{array}}\end{pmatrix}$$
$$=(0.2907,0.8267).$$

Theorem 4.3: Let $\rho_j=\left(\mu_{\rho_j},\eta_{\rho_j}\right)$ *(j= 1,2,…n)* are PFVs, then their aggregated value by applying PFWG operator is also a PFV , and

$$PFWG_\omega(\rho_1,\rho_2,...,\rho_n)=\left(\prod_{j=1}^{n}\mu_{\rho_j}^{\omega_j},\sqrt{1-\prod_{j=1}^{n}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}\right) \qquad (11)$$

and also the weighted vector of $\rho_j\,(j=1,2,...,n)$ is $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ with some conditions $\omega_j\in[0,1]$ and $\sum_{j=1}^{n}\omega_j=1$.

Proof: By mathematical induction we can prove that equation (11) holds for all *n*. First we show that equation (11) holds for *n= 2*, since

So
$$\rho_1^{\omega_1}=\left(\mu_{\rho_1}^{\omega_1},\sqrt{1-\left(1-\eta_{\rho_1}^2\right)^{\omega_1}}\right)$$
$$\rho_2^{\omega_2}=\left(\mu_{\rho_2}^{\omega_2},\sqrt{1-\left(1-\eta_{\rho_2}^2\right)^{\omega_2}}\right)$$

$$\rho_1^{\omega_1}\oplus\rho_2^{\omega_2}$$
$$=\left(\mu_{\rho_1}^{\omega_1},\sqrt{1-\left(1-\eta_{\rho_1}^2\right)^{\omega_1}}\right)\otimes$$
$$\left(\mu_{\rho_2}^{\omega_2},\sqrt{1-\left(1-\eta_{\rho_2}^2\right)^{\omega_2}}\right)$$
$$=\begin{pmatrix}\mu_{\rho_1}^{\omega_1}\mu_{\rho_2}^{\omega_2},\\ \sqrt{\begin{array}{c}\left(\sqrt{1-\left(1-\eta_{\rho_1}^2\right)^{\omega_1}}\right)^2+\left(\sqrt{1-\left(1-\eta_{\rho_2}^2\right)^{\omega_2}}\right)^2\\ -\left(\sqrt{1-\left(1-\eta_{\rho_1}^2\right)^{\omega_1}}\right)^2\left(\sqrt{1-\left(1-\eta_{\rho_2}^2\right)^{\omega_2}}\right)^2\end{array}}\end{pmatrix}$$
$$=\left(\prod_{j=1}^{2}\mu_{\rho_j}^{\omega_j},\sqrt{1-\prod_{j=1}^{2}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}\right)$$

Thus equation (11) true for *n= 2*. Let us suppose that equation (11) true for *n=k*. Then we have

$$PFWG_\omega(\rho_1,\rho_2,...,\rho_k)=\left(\prod_{j=1}^{k}\mu_{\rho_j}^{\omega_j},\sqrt{1-\prod_{j=1}^{k}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}\right)$$

Now we show that equation (11) true for *n=k+1*.

$$PFWG_\omega(\rho_1,\rho_2,...,\rho_{k+1})$$
$$=\rho_1^{\omega_1}\otimes\rho_2^{\omega_2}\otimes...\otimes\rho_{k+1}^{\omega_{k+1}}$$
$$=\left(\prod_{j=1}^{k}\mu_{\rho_j}^{\omega_j},\sqrt{1-\prod_{j=1}^{k}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}\right)\otimes$$
$$\left(\left(\mu_{\rho_{k+1}}\right)^{\omega_{k+1}},\sqrt{1-\left(1-\eta_{\rho_{k+1}}^2\right)^{\omega_{k+1}}}\right)$$
$$=\begin{pmatrix}\prod_{j=1}^{k}\mu_{\rho_j}^{\omega_j}.\left(\mu_{\rho_{k+1}}\right)^{\omega_{k+1}},\\ \sqrt{\begin{array}{c}1-\prod_{j=1}^{k}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}+1-\left(1-\eta_{\rho_{k+1}}^2\right)^{\omega_{k+1}}-\\ \left(1-\prod_{j=1}^{k}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}\right)\left(1-\left(1-\eta_{\rho_{k+1}}^2\right)^{\omega_{k+1}}\right)\end{array}}\end{pmatrix}$$
$$=\left(\prod_{j=1}^{k+1}\mu_{\rho_j}^{\omega_j},\sqrt{1-\prod_{j=1}^{k+1}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}\right)$$

Hence equation (11) holds for *n = k+1*. Thus equation (11) holds for all *n*

Example 4.4: Let $\rho_1=(0.4,0.8)$, $\rho_2=(0.5,0.7)$,

$\rho_3=(0.6,0.7)$, $\rho_4=(0.7,0.4)$ be four PFVs, and their weighted vector is $\omega=(0.1,0.2,0.3,0.4)^T$, then if

we apply the PFWG operator we get the Pythagorean fuzzy

value. Thus

$$PFWG_\omega(\rho_1,\rho_2,\rho_3,\rho_4)$$

$$=\left(\prod_{j=1}^{4}\mu_{\rho_j}^{\omega_j},\sqrt{1-\prod_{j=1}^{4}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}\right)$$

$$=\left(\begin{array}{c}(0.4)^{0.1}\times(0.5)^{0.2}\times(0.6)^{0.3}\times(0.7)^{0.4},\\ \sqrt{\begin{array}{c}1-(1-0.64)^{0.1}(1-0.49)^{0.2}\\(1-0.49)^{0.3}(1-0.16)^{0.4}\end{array}}\end{array}\right)$$

$$=(0.5907,0.6315).$$

Theorem 4.5: Let $\rho_j=\left(\mu_{\rho_j},\eta_{\rho_j}\right)(j=1,2,3,...,n)$ be The PFVsand the weighted vector of $\rho_j\,(j=1,2,...,n)$ is $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ with some conditions $\omega_j\in[0,1]$ and $\sum_{j=1}^{n}\omega_j=1.$ If $\rho_j\,(j=1,2,...,n)$ are mathematically equal. Then

$$PFWG_\omega(\rho_1,\rho_2,...,\rho_n)=\rho$$

Proof: As we know that

$$PFWG_\omega(\rho_1,\rho_2,...,\rho_n)=\rho_1^{\omega_1}\otimes\rho_2^{\omega_2}\otimes...\oplus\rho_n^{\omega_n}.$$

Let $\rho_j\,(j=1,2,3,...,n)=\rho$, then

$$PFWG_\omega(\rho_1,\rho_2,...,\rho_n)=\rho^{\omega_1}\otimes\rho^{\omega_2}\otimes...\oplus\rho^{\omega_n}$$

$$=(\rho)^{\sum_{j=1}^{n}\omega_j}$$

$$=\rho.$$

Example 4.6: Let $\rho_1=(0.4,0.8)$, $\rho_2=(0.4,0.8)$, $\rho_3=(0.4,0.8)$, $\rho_4=(0.4,0.8)$ be four PFVs, and their weighted vector is $\omega=(0.1,0.2,0.3,0.4)^T$. If we apply the PFWG operator we get the Pythagorean fuzzy valve.

$$PFWG_\omega(\rho_1,\rho_2,\rho_3,\rho_4)$$

$$=\left(\prod_{j=1}^{4}\mu_{\rho_j}^{\omega_j},\sqrt{1-\prod_{j=1}^{4}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}\right)$$

$$=\left(\begin{array}{c}(0.4)^{0.1}\times(0.4)^{0.2}\times(0.4)^{0.3}\times(0.4)^{0.4},\\ \sqrt{\begin{array}{c}1-(1-0.64)^{0.1}(1-0.64)^{0.2}\\(1-0.64)^{0.3}(1-0.64)^{0.4}\end{array}}\end{array}\right)$$

$$=(0.4,0.8).$$

Theorem 4.7: Let $\rho_j=\left(\mu_{\rho_j},\eta_{\rho_j}\right)\,(j=1,2,...,n)$ be the *PFVs* and let the weighted vector of $\rho_j\,(j=1,2,...,n)$ is $\omega=(\omega_1,\omega_2,...,\omega_n)^T$ such that

$\omega_j\in[0,1]$ and $\sum_{j=1}^{n}\omega_j=1.$ If

$$\rho^-=\left(\min_j\left(\mu_{\rho_j}\right),\max_j\left(\eta_{\rho_j}\right)\right),$$

$$\rho^+=\left(\max_j\left(\mu_{\rho_j}\right),\min_j\left(\eta_{\rho_j}\right)\right).$$

Then

$$\rho^-\le PFWG_\omega(\rho_1,\rho_2,...,\rho_n)\le\rho^+,\text{ for all }\omega. \quad (13)$$

Proof : As we know that

$$\min_j\left(\mu_{\rho_j}\right)\le\mu_{\rho_j}\le\max_j\left(\mu_{\rho_j}\right), \quad (14)$$

$$\min_j\left(\eta_{\rho_j}\right)\le\eta_{\rho_j}\le\max_j\left(\eta_{\rho_j}\right) \quad (15)$$

From equation (14), we have

$$\begin{aligned}&\Leftrightarrow\min_j\left(\mu_{\rho_j}\right)\le\mu_{\rho_j}\le\max_j\left(\mu_{\rho_j}\right)\\&\Leftrightarrow\min_j\left(\mu_{\rho_j}\right)^{\omega_j}\le\mu_{\rho_j}^{\omega_j}\le\max_j\left(\mu_{\rho_j}\right)^{\omega_j}\\&\Leftrightarrow\prod_{j=1}^{n}\min_j\left(\mu_{\rho_j}\right)^{\omega_j}\le\prod_{j=1}^{n}\mu_{\rho_j}^{\omega_j}\le\prod_{j=1}^{n}\max_j\left(\mu_{\rho_j}\right)^{\omega_j}\\&\Leftrightarrow\min_j\left(\mu_{\rho_j}\right)\le\prod_{j=1}^{n}\mu_{\rho_j}^{\omega_j}\le\max_j\left(\mu_{\rho_j}\right)\end{aligned} \quad (16)$$

Now from equation (15), we have

$$\Leftrightarrow\sqrt{1-\max_j\left(\eta_{\rho_j}\right)^2}\le\sqrt{1-\eta_{\rho_j}^2}\le\sqrt{1-\min_j\left(\eta_{\rho_j}\right)^2}$$

$$\Leftrightarrow\sqrt{\prod_{j=1}^{n}\left(1-\max_j\left(\eta_{\rho_j}\right)^2\right)^{\omega_j}}\le\sqrt{\prod_{j=1}^{n}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}$$

$$\leq \sqrt{\prod_{j=1}^{n}\left(1-\min_j\left(\eta_{\rho_j}\right)^2\right)^{\omega_j}}$$

$$\Leftrightarrow \sqrt{1-\max_j\left(\eta_{\rho_j}\right)^2} \leq \sqrt{\prod_{j=1}^{n}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}$$

$$\leq \sqrt{1-\min_j\left(\eta_{\rho_j}\right)^2}$$

$$\Leftrightarrow \min_j\left(\eta_{\rho_j}\right) \leq \sqrt{1-\prod_{j=1}^{n}\left(1-\eta_{\rho_j}^2\right)^{v_j}} \leq \max_j\left(\eta_{\rho_j}\right) \quad (17)$$

Let $PFWG_w(\rho_1,\rho_2,...,\rho_n)=\rho=\left(\mu_\rho,\eta_\rho\right)$, then,

$$S(\rho)=\mu_\rho^2-\eta_\rho^2 \leq \max{}_j\left(\mu_\rho\right)^2-\min{}_j\left(\eta_\rho\right)^2$$
$$=S\left(\rho^+\right)$$

Thus $S(\rho)\leq S\left(\rho^+\right)$. Again

$$S(\rho)=\mu_\rho^2-\eta_\rho^2 \geq \min{}_j\left(\mu_\rho\right)^2-\max{}_j\left(\eta_\rho\right)^2$$
$$=S\left(\rho^-\right).$$

Thus $S(\rho)\geq S\left(\rho^-\right)$. If $S(\rho)<S\left(\rho^+\right)$ and $S(\rho)>S\left(\rho^-\right)$. Then

$$\rho^- < PFWG_w(\rho_1,\rho_2,...,\rho_n) < \rho^+ \quad (18)$$

If $S(\rho) = S\left(\rho^+\right)$, then

$$\Leftrightarrow \mu_\rho^2-\eta_\rho^2=\max_j\left(\mu_{\rho_j}\right)^2-\min_j\left(\eta_{\rho_j}\right)^2$$

$$\Leftrightarrow \mu_\rho^2=\max_j\left(\mu_{\rho_j}\right)^2, \eta_\rho^2=\min_j\left(\eta_{\rho_j}\right)^2$$

$$\Leftrightarrow \mu_\rho=\max_j\left(\mu_{\rho_j}\right), \eta_\rho=\min_j\left(\eta_{\rho_j}\right).$$

Since

$$H(\rho)=\mu_\rho^2+\eta_\rho^2=\max{}_j\left(\mu_{\rho_j}\right)^2+\min{}_j\left(\eta_{\rho_j}\right)^2=H\left(\rho^+\right).$$

Thus

$$PFWG_w(\rho_1,\rho_2,...,\rho_n)=\rho^+ \quad (19)$$

If $S(\rho)=S\left(\rho^-\right)$, then

$$\Leftrightarrow \mu_\rho^2-\eta_\rho^2=\min_j\left(\eta_{\rho_j}\right)^2-\max_j\left(\mu_{\rho_j}\right)^2$$

$$\Leftrightarrow \mu_\rho^2=\min_j\left(\eta_{\rho_j}\right)^2, \nu_\rho^2=\max_j\left(\mu_{\rho_j}\right)^2$$

$$\Leftrightarrow \mu_\rho=\min_j\left(\eta_{\rho_j}\right), \eta_\rho=\max_j\left(\mu_{\rho_j}\right).$$

Since

$$H(\rho)=\mu_\rho^2+\eta_\rho^2=\min{}_j\left(\eta_{\rho_j}\right)^2+\max{}_j\left(\mu_{\rho_j}\right)^2$$
$$=H\left(\rho^-\right).$$

Thus

$$PFWG_\omega(\rho_1,\rho_2,...,\rho_n)=\rho^- \quad (20)$$

Thus from equation (18) to (20), we have

$\rho^- \leq PFWG_\omega(\rho_1,\rho_2,...,\rho_n)\leq\rho^+$, for all ω.

Theorem 4.8: Let $\rho_j=\left(\mu_{\rho_j},\eta_{\rho_j}\right)$ $(j=1,2,3,...,n)$

And $\rho_j^*=\left(\mu_{\rho_j^*},\eta_{\rho_j^*}\right)$ $(j=1,2,3,...,n)$ be the two collection of *PFVs*. If $\mu_{\rho_j}\leq\mu_{\rho_j^*}$ and $\eta_{\rho_j}\geq\eta_{\rho_j^*}$.

Then

$$PFWG_\omega(\rho_1,\rho_2,...,\rho_n)\leq PFWG_\omega\left(\rho_1^*,\rho_2^*,...,\rho_n^*\right) \quad (21)$$

Proof: Since, $\mu_{\rho_j}\leq\mu_{\rho_j^*}$ *and* $\eta_{\rho_j}\geq\eta_{\rho_j^*}$. and Then

$$\begin{aligned}&\Leftrightarrow \mu_{\rho_j}^{\omega_j}\leq\mu_{\rho_j^*}^{\omega_j}\\ &\Leftrightarrow \prod_{j=1}^{n}\mu_{\rho_j}^{\omega_j}\leq\prod_{j=1}^{n}\mu_{\rho_j^*}^{\omega_j}\end{aligned} \quad (22)$$

Now by using the non-membership function we have

$$\Leftrightarrow 1-\eta_{\rho_j}^2\leq 1-\eta_{\rho_j^*}^2$$

$$\Leftrightarrow \sqrt{\prod_{j=1}^{n}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}} \leq \sqrt{\prod_{j=1}^{n}\left(1-\eta_{\rho_j^*}^2\right)^{\omega_j}} \quad (23)$$

$$\Leftrightarrow \sqrt{1-\prod_{j=1}^{n}\left(1-\eta_{\rho_j^*}^2\right)^{\omega_j}} \leq \sqrt{1-\prod_{j=1}^{n}\left(1-\eta_{\rho_j}^2\right)^{\omega_j}}$$

Let

$$\rho=PFWG_\omega(\rho_1,\rho_2,...,\rho_n) \quad (24)$$

and

$$\rho^*=PFWG_\omega\left(\rho_1^*,\rho_2^*,...,\rho_n^*\right) \quad (25)$$

Then from equation (21) we have, $S(\rho) \le S(\rho^*)$. If $S(\rho) < S(\rho^*)$, then

$$PFWG_\omega(\rho_1, \rho_2, ..., \rho_n) < PFWG_\omega(\rho_1^*, \rho_2^*, ..., \rho_n^*) \quad (26)$$

If $S(\rho) = S(\rho^*)$, then

$$\Leftrightarrow \mu_\rho^2 - \eta_\rho^2 = \mu_{\rho^*}^2 - \eta_{\rho^*}^2$$
$$\Leftrightarrow \mu_\rho^2 = \mu_{\rho^*}^2, \eta_\rho^2 = \eta_{\rho^*}^2$$
$$\Leftrightarrow \mu_\rho = \mu_{\rho^*}, \eta_\rho = \eta_{\rho^*}.$$

Since $H(\rho) = \mu_\rho^2 + \eta_\rho^2 = \mu_{\rho^*}^2 + \eta_{\rho^*}^2 = H(\rho^*)$. Thus

$$PFWG_\omega(\rho_1, \rho_2, ..., \rho_n) = PFWG_\omega(\rho_1^*, \rho_2^*, ..., \rho_n^*) \quad (27)$$

Thus from equation (26) and (27), we have

$$PFWG_\omega(\rho_1, \rho_2, ..., \rho_n) \le PFWG_\omega(\rho_1^*, \rho_2^*, ..., \rho_n^*)$$

Example: 4.9: Let $\rho_1 = (0.4, 0.6)$, $\rho_2 = (0.5, 0.7)$, $\rho_3 = (0.3, 0.8), \rho_4 = (0.2, 0.9)$, ${\rho^*}_1 = (0.7, 0.5)$ ${\rho^*}_2 = (0.8, 0.3), {\rho^*}_3 = (0.6, 0.5), {\rho^*}_4 = (0.5, 0.5)$ and also $\omega = (0.1, 0.2, 0.3, 0.4)$.

Now using the PFWG operator we get the following result.

$$PFWG_w(\rho_1, \rho_2, \rho_3, \rho_4)$$
$$= \left(\prod_{j=1}^{4} \mu_{\rho_j}^{\omega_j}, \sqrt{1 - \prod_{j=1}^{4} \left(1 - \eta_{\rho_j}^2\right)^{\omega_j}} \right)$$
$$= \left(\begin{array}{c} (0.4)^{0.1} \times (0.5)^{0.2} \times (0.3)^{0.3} \times (0.2)^{0.4}, \\ \sqrt{\begin{array}{c} 1 - (1-0.36)^{0.1}(1-0.49)^{0.2} \\ (1-0.64)^{0.3}(1-0.81)^{0.4} \end{array}} \end{array} \right)$$
$$= (0.2907, 0.8267).$$

Again

$$PFWG_w({\rho^*}_1, {\rho^*}_2, {\rho^*}_3, {\rho^*}_4)$$

$$= \left(\prod_{j=1}^{4} \mu_{{\rho^*}_j}^{\omega_j}, \sqrt{1 - \prod_{j=1}^{4} \left(1 - \eta_{{\rho^*}_j}^2\right)^{\omega_j}} \right)$$
$$= \left(\begin{array}{c} (0.7)^{0.1} \times (0.8)^{0.2} \times (0.6)^{0.3} \times (0.5)^{0.4}, \\ \sqrt{\begin{array}{c} 1 - (1-0.25)^{0.1}(1-0.09)^{0.2} \\ (1-0.25)^{0.3}(1-0.25)^{0.4} \end{array}} \end{array} \right)$$
$$= (0.6000, 0.4695).$$

5. An Application of the PFWG Operator to MAGDM Problem

In this section, we discuss an application of the *PFWG* operator to MADM. Now we are using Pythagorean fuzzy information to develop the MADM.

***Algorithm*:** Let M = {M_1, M_2, M_3,…,M_n} be a finite set of n alternatives, and suppose O = {O_1, O_2, O_3,…,O_m} is a finite set of m attributes , and $D = \{D_1, ..., D_k\}$ be the set of k experts.

Let $\omega = (\omega_1, \omega_2, ..., \omega_m)^T$ be the weighted vector of the attributes O_j $(j = 1, ..., m)$, also $\omega_j \in [0,1]$ and $\sum_{j=1}^{m} \omega_j = 1$, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_k)^T$ be the weighted vector of the D^s $(s = 1, ..., k)$, also $\lambda_s \in [0,1]$ and $\sum_{s=1}^{k} \lambda_s = 1.$

This method have the following steps.

Step 1: Construct the Pythagorean fuzzy decision matrices $D^s = \left[d_{ij}^{(s)}\right]_{n\times m}$ $(s = 1, 2, ..., k)$ for decision. If the criteria have two types, one is benefit criteria and other is cost criteria, then the decision maker transform the Pythagorean fuzzy decision matrix, $D^s = \left[d_{ij}^{(s)}\right]_{n\times m}$, into the normalized Pythagorean fuzzy decision matrix,

$R^s = \left[r_{ij}^{(s)}\right]_{n\times m}$, where

$$r_{ij}^{(s)} = \begin{cases} d_{ij}, \text{ for benefit criteria } O_j \\ d_{ij}^c, \text{ for cost criteria } O_j, \end{cases} \quad (j = 1, 2, ..., m),$$

where d_{ij}^c be the complement of d_{ij}. If all the criteria have the same type, then there is no need of normalization.

Step 2: In this step we are going to apply the PFWG operator tocombined the entire individual PFDMS $R^s = \left[r_{ij}^{(s)}\right]_{n\times m}$ $(s = 1, 2, ..., k)$ into the collective PFDM $R = \left[r_{ij}\right]_{n\times m}$, with condition $r_{ij}=\left(\mu_{ij},\nu_{ij}\right)\begin{pmatrix} i=1,2,..,n, \\ j=1,2,...,m \end{pmatrix}$.

Step 3: Aggregate all the preference values $r_{ij}=\left(\mu_{ij},\nu_{ij}\right)\begin{pmatrix} i=1,2,..,n, \\ j=1,2,...,m \end{pmatrix}$. by using the PFWG operatorand get the overall preference value $r_i\ (i=1,2,3,...n)$ analogous to the alternative M_i $(i=1,2,3,...n)$.

Step 4: In this step we determine the scores of $r_i\,(i=1,2,3,...n)$. If there is difference between two or more than two score functions then we have must to calculate the accuracy degrees.

Step 5: In this step we arrange the score values of each alternative by descending order and chose the best alternative by maximum value of score function.

Example 5.1: The plant location selection problem.Suppose a company is searching a geographical place for new plantation. The company wants to plant these plants in the following best conditions, such as, low cost, best climatic conditions, having safety from surrounding. There are many factors that must be deliberated while choosing a appropriate place for a plant, now we are going to choose the most common and important four attributes.

1. O_1 : Expert workers,
2. O_2 : Transport facilities,
3. O_3 : Investment cost,
4. O_4 : Expansion possibility.

where O_1, $O_3$are cost criteria, and O_2, O_4 are benefit criteria. After preliminary screening, five locations M_1,M_2,M_3,M_4,M_5 are selected for additional estimation. A group of three selection makers, $D^k\left(k=1,2,3\right)$is choosing to choose a best option out of these five places. Let $\lambda=\left(0.2,0.3,0.5\right)^T$ is the weighted vector of $D^k\left(k=1,2,3\right)$ and $\omega=\left(0.1,0.2,0.3,0.4\right)^T$ is the weighted vector of $O_j\left(j=1,...,4\right)$.

Step 1: The decision makers give his decision in the following tables.

Table 1: Pythagorean fuzzy decision matrix D_1

	O_1	O_2	O_3	O_4
M_1	(0.8,0.3)	(0.8,0.4)	(0.7,0.4)	(0.6,0.5)
M_2	(0.7,0.3)	(0.8,0.4)	(0.6,0.5)	(0.7,0.3)
M_3	(0.5,0.5)	(0.6,0.4)	(0.7,0.4)	(0.8,0.3)
M_4	(0.6,0.5)	(0.7,0.4)	(0.8,0.4)	(0.8,0.5)
M_5	(0.6,0.6)	(0.7,0.3)	(0.8,0.3)	(0.8,0.5)

Table 2: Pythagorean fuzzy decision matrix D_2

	O_1	O_2	O_3	O_4
M_1	(0.2,0.8)	(0.7,0.4)	(0.4,0.6)	(0.6,0.5)
M_2	(0.2,0.8)	(0.8,0.4)	(0.5,0.6)	(0.7,0.3)
M_3	(0.5,0.6)	(0.7,0.3)	(0.4,0.6)	(0.8,0.3)
M_4	(0.3,0.7)	(0.6,0.4)	(0.4,0.7)	(0.8,0.5)
M_5	(0.4,0.6)	(0.8,0.2)	(0.3,0.8)	(0.8,0.4)

Table 3: Pythagorean Fuzzy Decision Matrix D_3

	O_1	O_2	O_3	O_4
M_1	(0.3,0.7)	(0.6,0.4)	(0.4,0.6)	(0.6,0.5)
M_2	(0.3,0.8)	(0.7,0.4)	(0.3,0.8)	(0.9,0.2)
M_3	(0.5,0.7)	(0.6,0.5)	(0.4,0.7)	(0.8,0.3)
M_4	(0.4,0.7)	(0.8,0.4)	(0.1,0.9)	(0.7,0.5)
M_5	(0.5,0.6)	(0.9,0.2)	(0.2,0.8)	(0.8,0.2)

Table 4: Normalize PFDM R_1

	O_1	O_2	O_3	O_4
M_1	(0.8,0.3)	(0.8,0.4)	(0.7,0.4)	(0.6,0.5)
M_2	(0.7,0.3)	(0.8,0.4)	(0.6,0.5)	(0.7,0.3)
M_3	(0.5,0.5)	(0.6,0.4)	(0.7,0.4)	(0.8,0.3)
M_4	(0.6,0.5)	(0.7,0.4)	(0.8,0.4)	(0.8,0.5)
M_5	(0.6,0.6)	(0.7,0.3)	(0.8,0.3)	(0.8,0.5)

Table 5: Normalize PFDM R_2

	O_1	O_2	O_3	O_4
M_1	(0.8,0.2)	(0.7,0.4)	(0.6,0.4)	(0.6,0.5)
M_2	(0.8,0.2)	(0.8,0.4)	(0.6,0.5)	(0.7,0.3)
M_3	(0.6,0.5)	(0.7,0.3)	(0.6,0.4)	(0.8,0.3)
M_4	(0.7,0.3)	(0.6,0.4)	(0.7,0.4)	(0.8,0.5)
M_5	(0.6,0.4)	(0.8,0.2)	(0.8,0.3)	(0.8,0.4)

Table 6: Normalize PFDM R_3

	O_1	O_2	O_3	O_4
M_1	(0.7,0.3)	(0.6,0.4)	(0.6,0.4)	(0.6,0.5)
M_2	(0.8,0.3)	(0.7,0.4)	(0.8,0.3)	(0.9,0.2)
M_3	(0.7,0.5)	(0.6,0.5)	(0.7,0.4)	(0.8,0.3)
M_4	(0.7,0.4)	(0.8,0.4)	(0.9,0.1)	(0.7,0.5)
M_5	(0.6,0.5)	(0.9,0.2)	(0.8,0.2)	(0.8,0.2)

Step 2: Apply the PFWG operator to collective all the normalized individual Pythagorean fuzzy decision matrices, $R^s=\left[r_{ij}^{(s)}\right]_{n\times m}$ into the collective PFDM $R=\left[r_{ij}\right]_{n\times m}$.

Table 7: Collective PFDM R

	O_1	O_2	O_3	O_4
M_1	(0.7,0.3)	(0.7,0.4)	(0.7,0.4)	(0.7,0.5)
M_2	(0.8,0.3)	(0.7,0.4)	(0.7,0.4)	(0.8,0.3)
M_3	(0.6,0.5)	(0.7,0.4)	(0.7,0.4)	(0.8,0.3)
M_4	(0.7,0.4)	(0.7,0.4)	(0.8,0.3)	(0.7,0.5)
M_5	(0.6,0.5)	(0.8,0.2)	(0.8,0.3)	(0.8,0.3)

Step 3: In this step we aggregate all the preference values r_{ij} $(i=1,2,...5,\ j=1,...,4)$ by using the PFWG operator and get the overall preference value r_i $(i=1,2,3,4,5)$ analogous to the alternative M_i $(i=1,...,5)$

$$r_1=(0.700,0.436), r_2=(0.748,0.354), r_3=(0.727,0.377),$$
$$r_4=(0.728,0.421), r_5=(0.777,0.312)$$

Step 4: In this step we determine the scores of r_i $(i=1,...,5)$.

$$S(r_1) = (0.700)^2 - (0.436)^2 = 0.299$$
$$S(r_2) = (0.748)^2 - (0.354)^2 = 0.434$$
$$S(r_3) = (0.727)^2 - (0.377)^2 = 0.386$$
$$S(r_4) = (0.728)^2 - (0.421)^2 = 0.352$$
$$S(r_5) = (0.777)^2 - (0.312)^2 = 0.506$$

Step 5: Now we arrange the score function of each alternative in the form of descendent order and chose the best alternative by maximum value of score function.

$$r_1 \leq_L r_4 \leq_L r_3 \leq_L r_2 \leq_L r_5.$$

Then

$M_5 > M_2 > M_3 > M_4 > M_1$. Since M_5 has the highest value. Thus M_5 is the best location among the stated locations for a company to plant the plants.

6. Conclusions

In this study, we have developed PFWG operator. We have explored different properties of this proposed operator. We have also utilized PFWG operator to multiple attribute decision making based on Pythagorean fuzzy information

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