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MHD Boundary Layer Flow of Micropolar Fluids due to Porous Shrinking Surface with Viscous dissipation and Radiation

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ABSTRACT

The mathematical analysis and numerical solution for the flow of micropolar fluids owing to shrinking boundary is considered in the presence of magnetic field and thermal radiation. The parametric study of the problem demonstrates the effects of magnetic field, suction, micropolar material parameter and thermal radiation on velocity, microrotation and temperature. The mathematical model of the problem is transformed to non-dimensional form to obtain numerical solution. The results have been obtained for several representative values of the material parameters d_1, d_2 and d_3, heat source parameter λ and magnetic parameter M, suction/injection parameter S, Eckert number E_c, Radiation parameter R_n and Prandtl number P_r . The flow speed and microrotation are slowed with incremented inputs of micropolar parameter d_1. The fluid temperature increases with radiation parameter but it diminishes against suction.

Keywords: *Micropolar fluids, Magnetic field, Heat source, Radiation, Prandtl number*

1. Introduction

The micropolar fluid theory pioneered by Eringen [1] presents relatively a new research field. This model besides the generalization of Navier-Stokes model takes into account the conservation of angular momentum due to local micromotion of the fluid particles. Baag et al. [2] obtained numerical solution for magnetohydrodynamic (MHD) flow of micropolar fluids near stagnation point on vertical surface with chemical reaction and heat source. Takhar et al. [3] solved numerically the flow and heat transfer for micropolar fluids due to porous disks. The heat and mass transfer for electrically conducting micropolar fluids over a stretching surface has been examined by Abo-Eldahab and El-Aziz [4]. Sajjad et al. [5] investigated hydro-magnetic micropolar fluid flow between two parallel plates, the lower plate is stretching. Barik and Dash [6] studied the flow of peristaltic motion of micropolar fluids in a two-dimensional channel and through a porous medium. Vimala and Omega [7] analyzed the 2-dimensional and steady laminar flow of a micropolar fluids in permeable channel. The magnetohyderodynamic viscous flow of micropolar fluid due to shrinking boundary has been solved numerically by Shafique [8]. Veena [9] discussed the effect of change in shape and size of micro molecules of a micro polar fluid on the variation of pressure and load capacity in a squeeze film bounded by a rigid plate.

Latterly, heat transfer in micropolar fluids was also discussed by Eringen [10]. Ahmad et al. [11] investigated unsteady blood flow having micropolar fluid properties with heat source through parallel plates channel under the influence of a uniform transverse magnetic field. Khilap and Manoj [12] analyzed the fluid flow and heat transfer characteristics occurring throughout the melting process over a moving boundary surface in micropolar fluid with thermal radiation. Waqas et al. [13] worked on mathematical analysis and numerical solution for micropolar fluids flow due to a shrinking porous surface in the presence of magnetic field and thermal radiation. Waqas et al. [14] studied the micropolar fluids near the stagnation point flow of electrically conducting due to a surface with the boundary in motion (stretching/shrinking).

The fluid flows under the effect of magnetic field in the presence of heat source and radiation is important. Khalid et al. [15] considered MHD fluid flow of thermal radiation and viscous dissipation due to porous shrinking sheet. Abdel-Rahman [16] discussed the effect of magnetohydronamic and focused on thin films for study in unsteady micropolar fluid. Asghar et al. [17] investigated the effects of Hall current and heat transfer on flow due to a pull of eccentric rotating disks. Mohyuddin and Goetz [18] studied resonance behavior of viscoelastic fluid in Poiseuille flow in the presence of a transversal magnetic field.

In the present study, we considered micropolar fluids flow due to a shrinking porous surface in the presence of magnetic field and thermal radiation to extend the work of Jeena [19]. In the previous work, Newtonian fluid is considered but most of the practical situations involve non Newtonian fluids. Micropolar fluids theory provides substantial generalization to the Navier-Stokes model. Thus our interest aroused to present dynamics of micropolar fluids for the problem considered. In addition to the velocity vector, a new quantity microrotation and four coefficients of viscosity are taken in to account. It resulted an additional equation on the basis of angular momentum. If microrotation vector ω and vortex viscosity k are omitted, the problem is reduced to Newtonian fluid flow. The numerical solution of the problem has been

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sought to examine the nature of fluid flow, microrotation and heat transfer. The flow with shrinking surface and the effect of material constants and radiative heat source give new dimension to the problem.

2. Methodology

Consider micropolar fluid flow towards the stagnation point on a porous stretching surface**.** The fluid is incompressible and electrically conducting. The flow is steady and two-dimensional. The magnetic field of strength B_o is applied perpendicular to the surface that stretches or shrinks along x-axis. The horizontal component of velocity varies proportional to a specified distance *x*. The velocity of flow in the region exterior to the boundary layer is *U=cx*. The surface temperature is *T*. The temperature in the region exterior to the boundary layer is T_∞. The body couple is absent. The velocity vector is $\boldsymbol{V} = V(u, v)$ and spin vector is $\boldsymbol{\omega} = \boldsymbol{\omega}(0,0,\omega_3)$.

Under the above assumptions the equations, governing the problems are [1]:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (\mu + \kappa)\frac{\partial^2 u}{\partial y^2} + \kappa\frac{\partial \omega_3}{\partial y} - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$\gamma\frac{\partial^2 \boldsymbol{\omega}}{\partial y^2} - \kappa(\frac{\partial u}{\partial y} + 2\boldsymbol{\omega}) = \rho j\left(u\frac{\partial \boldsymbol{\omega}}{\partial x} + v\frac{\partial \boldsymbol{\omega}}{\partial y}\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{K}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0(T - T_\infty)}{\rho C_p} + \frac{\mu}{\rho C_p}(\frac{\partial u}{\partial y})^2 + \frac{16\alpha^*}{3\,\beta^*\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_0^2}{\rho C_p}u^2 \tag{4}$$

Here ρ is density, μ is dynamic viscosity, σ is the electrical conductivity, K is the thermal conductivity, C_p is the specific heat capacity at constant pressure, T_∞ is the stream temperature, Q_0 is the volumetric rate of heat generation, *k* and γ are additional viscosity coefficients for micropolar fluid and *j* is micro inertia, Stefan Boltzmann constant is α^* and Roseland mean absorption coefficient is denoted by β^*.

The boundary conditions are:

$$\left.\begin{array}{c} \omega_3(x, 0) = 0, u(x.0) = U_w = -cx, \\ v(x.0) = -v_w, T(x.0) = T_w, \\ \omega_3(x, \infty) = 0, u(x.\infty) = 0, \\ T(x.\infty) = T_\infty \end{array}\right\} \tag{5}$$

Here $c > 0\ (0 < c < 1)$ stands for shrinking of the sheet. T_w is the wall temperature, $v_w(v_w > 0)$ is a prescribed distribution of wall mass suction through porous sheet, κ is vortex viscosity. .

By using similarity transformations, the velocity components are described below in terms of the stream function ψ (x,y):

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x},\ \psi\,(x,y) = x\sqrt{cv}\,f(\eta)$$

$$\eta = y\sqrt{\frac{c}{v}}\quad,\quad u = xcf',\ v = -\sqrt{vc}f$$

$$\omega_3 = \sqrt{\frac{c}{v}}\,cxL(\eta)\,,\qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

Equation of continuity (1) is identically satisfied.

Substituting the above appropriate relation in equations (2), (3) and (4), we get:

$$(1 + d_1)f''' + d_1 L' - Mf' = f'^2 - ff'' \tag{6}$$

$$d_3 L'' + d_1 d_2(L - f'') = f'L - fL' \tag{7}$$

$$(4 + 3R_n) = \theta'' + 3R_n P_r\ (f\theta' + \lambda\,\theta + E_c f''^2 + ME_c f'^2) \tag{8}$$

and the boundary conditions (5) become:

$$\left.\begin{array}{c} f'(0) = -1,\ f(0) = S, L(0) = 0, \theta(0) = 1 \\ f'(\infty) = 0, L(\infty) = 0, \theta(\infty) = 1 \end{array}\right\} \tag{9}$$

Whereas $P_r = \frac{v}{\alpha}$ is Prandtl number, $E_c = \frac{U_w^2}{C_p\,(T_w - T_\infty)}$ is Eckert number, $\lambda = \frac{Q_0}{\rho\, C_p c}$ is heat source ($\lambda < 0$) or sink ($\lambda > 0$), $M = \frac{\sigma B_0^2}{\rho c}$ is magnetic parameter, $S = \frac{v_0}{\sqrt{cv}} > 0$ is mass suction parameter.

$$d_1 = \frac{\kappa}{\mu}, d_2 = \frac{\mu}{\rho j v}, d_3 = \frac{\gamma}{\rho j v}$$

are dimensionless material constants.

3. Results and Discussion

The non-linear differential Eqs. (6) to (9) have been solved numerically as such systems are difficult to be solved analytically [20, 21]. The higher order derivatives are reduced to set of first order differential equations to implement Runge Katta method with shooting technique as described in various studies [22, 23]. The resulting equations are coded in the environment of Mathematica software. Several computations have been made for viable ranges of the physical parameters involved in the study. The fixed values of parameters are taken arbitrarily as $M = 2$, $S = 2$, $P_r = 0.7$, $E_c = 0.1$, $\lambda = 0.1$ and $R_n = 0.1$. The results of the physical quantities namely tangential velocity f', micromotion L and temperature function $\theta(\eta)$ have been presented in graphical form. Fig. 1 indicates the effect the magnetic force field on velocity f'. It is observed that velocity decreases in magnitude with increase of *M.* It happens due to the incremented resistive force, known as Lorentz force which comes in to play during the interaction of electric and magnetic fields. The progressive suction parameter *S* decelerates the flow in horizontal direction and magnitude of f' depreciates against *S* as noticed in Fig. 2. However, the velocity f' increases in the magnitude with increase in the value of micropolar parameter d_1 as depicted in the fig. 3. Similarly, fig. 4 reveals that the micromotion function *L* reduces with d_1 because vortex viscosity is intensified in this situation and hence the micromotion is strengthened. Fig. 5 demonstrates the effect of Prandtl number P_r on heat function $\theta(\eta)$. It is noticed that higher inputs of P_r causes to decline the temperature curve. It can be justified by the fact that thermal diffusivity decreases with increments of P_r. Fig. 6 delineates the impact of heat source index $\lambda\,(\lambda > 0)$ on $\theta(\eta)$ and it is seen that fluid temperature rises directly with λ. However, temperature distribution decreases with increase of the values of suction parameter S as depicted in fig. 7.

Fig. 8 demonstrates that the temperature function $\theta(\eta)$ rises with increment of radiation parameter R_n because the radiative mode of heat transportations is enhanced. Eckert number E_c corresponds to thermal dissipation which results in conversion of mechanical energy into heat energy. The increments in E_c raise the temperature $\theta(\eta)$ as shown in fig. 9.

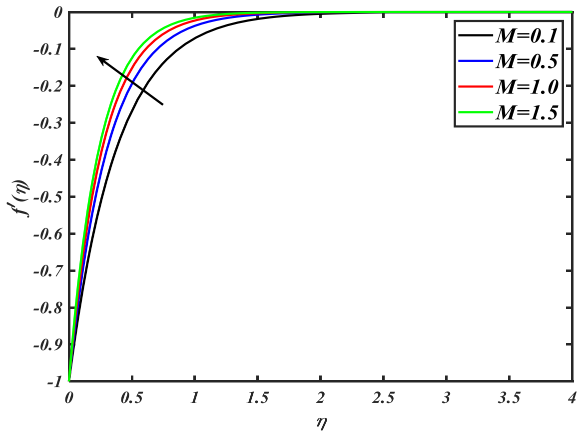

Fig. 1: The plot for curves of f' under the effect of magnetic parameter M.

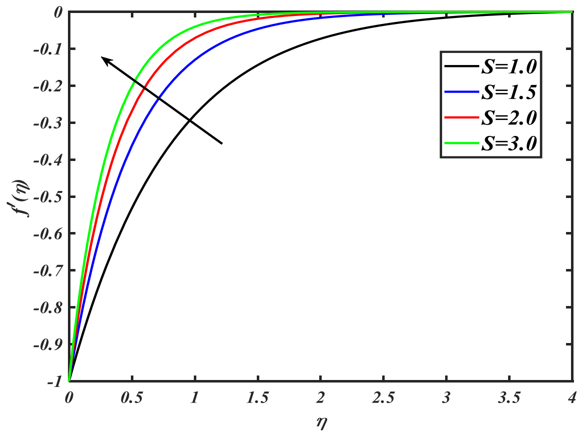

Fig. 2: The plot for curves of f' under the effect of suction parameter S.

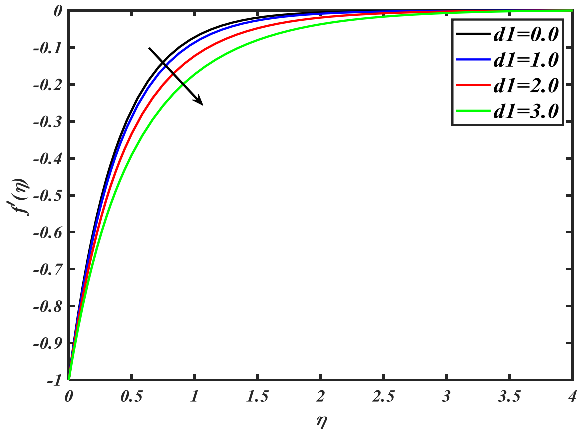

Fig. 3: The plot for curves of f' under the effect of micropolar parameter d_1.

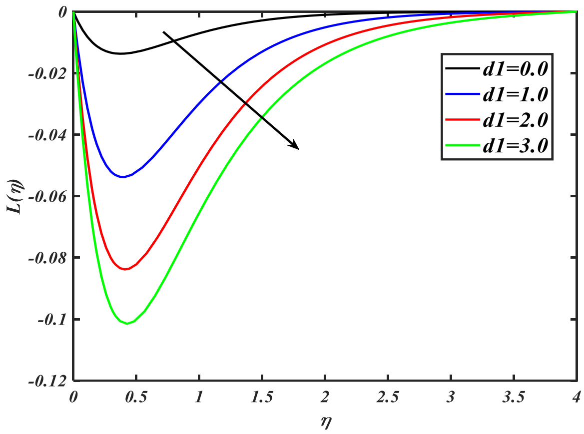

Fig. 4: The plot for curves of microrotation L under the effect of d_1.

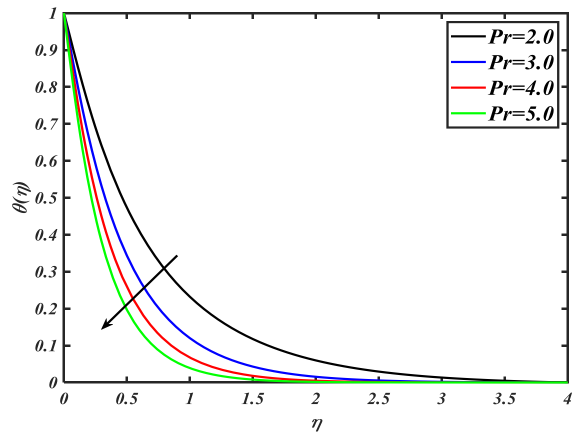

Fig. 5: The plot for curves of θ under the effect of Prandtl number P_r and R_n=0.1.

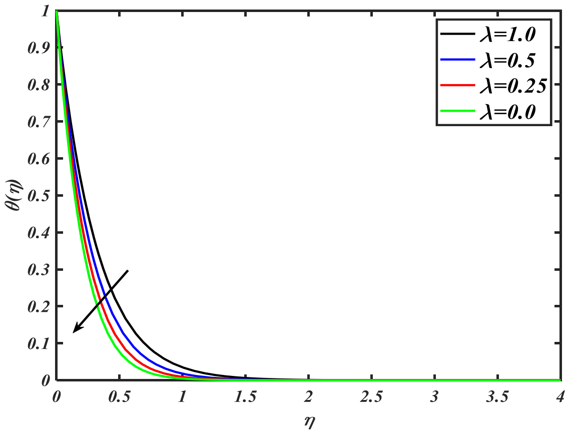

Fig. 6: The plot for curves of θ under the effect of λ.

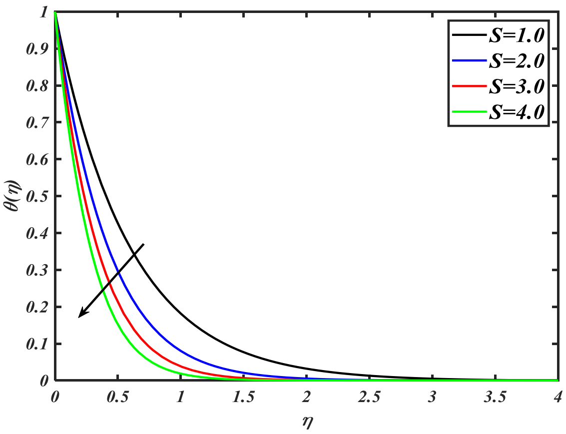

Fig. 7: The plot for curves of θ under the effect of suction parameter *S*.

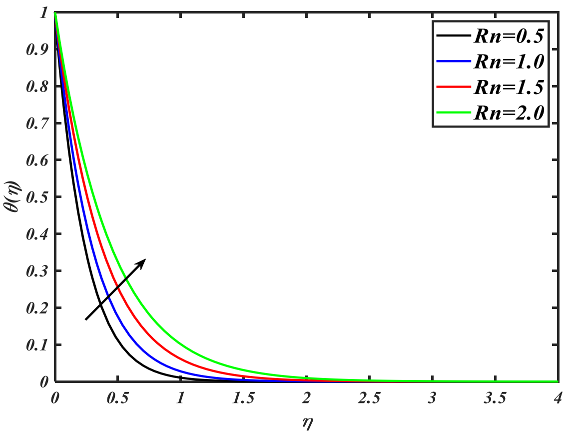

Fig. 8: The plot for curves of θ under the effect of radiation parameter *Rn*.

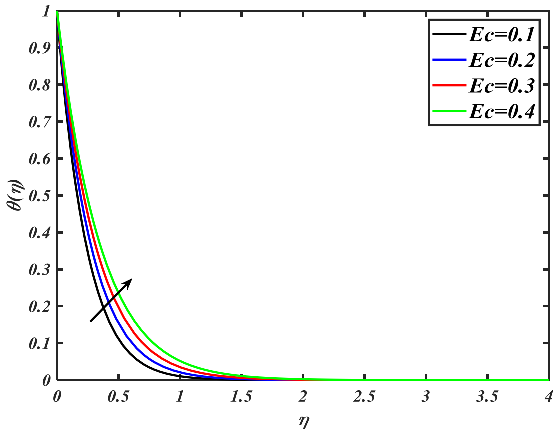

Fig. 9: The plot for curves of θ under the effect of Eckert number E_c.

4. Conclusions

This work examined the MHD boundary layer flow of micropolar fluids due to porous shrinking surface with viscous dissipation and radiation effects. The results of the physical quantities namely tangential velocity f', momentum of micromotion L and temperature function $\theta(\eta)$ have been computed for viable ranges of pertinent parameters of physical importance involved in the model of the problem.

- The velocity f' decreases in magnitude with increase of the magnetic force field and suction at the surface.
- The velocity f' and micromotion function L increase in the magnitude with increase in the value of micropolar parameter d_1.
- The heat function $\theta(\eta)$ decreases with increase of the values of Prandtl number P_r, heat source or sink index λ and suction parameter S.
- The temperature function $\theta(\eta)$ increases with increase in the values of radiation parameter R_n and Eckert number E_c.

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