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Construction of Generalized Neighbor Designs in Circular Blocks of Two Different Sizes

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ABSTRACT

Neighbor balanced designs are used in experiments where neighbor effects may arise. These designs ensure that treatment comparisons will be less affected by the neighbor effects. Experimenters always prefer minimal designs but minimal circular neighbor designs cannot be constructed for even v, where v is number of treatments to be compared. Therefore, for v even, minimal circular generalized neighbor designs (MGNDs) should be used which are considered to be the good alternate to the minimal neighbor designs. In this article, some generators are developed to obtain MCGNDs in which 3v/2 unordered pairs of distinct treatments appear twice as neighbors while others appear once. Catalogues of the designs provide readymade solutions to the experimenters and researchers. The proposed designs catalogues are also compiled from the generators developed in this study for v (even) ≤ 100.

Keywords: *Neighbor Effects, Neighbor Designs, Generalized Neighbor Designs, Robust to Neighbor Effects*

1. Introduction

There are many experiments where the performance of a treatment is also influenced by the treatments applied to its neighboring units. The influence of the treatments applied to neighboring units is called neighbor effect which is a major source of bias. In such experiments, neighbor balanced designs (NBDs) should be used because these designs ensure that treatments comparisons will be less affected by the neighbor effects. Among the NBDs, minimal NBD is considered to be most economical, therefore, experimenters prefer it. A design in which each pair of distinct treatments appears once in the adjacent plots of the same block is called minimal NBD. Ref. [1] used circular NBDs in virus research. Border plots [2] are constructed in circular NBDs. Using method of cyclic shifts, [3-6] constructed minimal NBDs for some cases of circular blocks. Minimal circular generalized neighbor designs (MCGNDs) should be used in the situations where minimal NBDs could not be constructed [7-13] for some cases. MCGNDs are more economical and better alternate to the NBDs if most of the unordered pairs of distinct treatments appear once as neighbors while remaining pairs appear twice. Ref. [14] presented list of CGNDs for blocks of sizes three.

In this article, MCGNDs are constructed in blocks of two different sizes using i sets of shifts for k_1 and two sets for k_2. In these designs, $3v/2$ unordered pairs of distinct treatments appear twice as neighbors while all others pairs appear once, where v is number of treatments.

2. Method of cyclic shifts

Method of cyclic shifts (Rule I) is described here to generate minimal CNBDs and MCGNDs. This meth15od was introduced by [15].

Let $S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$ be i sets of shifts, j = 1,2,…, i, u = 1,2,…, k-1, and $1 \le q_{ju} \le v$-1.

- If each of 1, 2, …, v-1 appears exactly once in S* then designs will be minimal CNBD.
- If each of 1, 2, …, v-1 appears either once and/or twice in S* then designs will be MCGND.

Where S* contains:

i. Each element of all sets S_j.
ii. Sum (mod v) of all elements in each set S_j.
iii. Complements of all elements in (i) and (ii), here complement of a is v-a.

Following is logic behind the method of cyclic shifts (Rule I) to construct MCGNDs in which $3v/2$ unordered pairs of distinct treatments appear twice as neighbors for $v = 2ik_1+4k_2-2$, where $k_1 > k_2$.

- A = [1, 2, …, m, m+1, m] or A = [1, 2, …, m, m+1, m+2] will provide required MCGNDs if sum of elements in A is divisible by v. If not, replace one or more elements with their complements to make the sum divisible by v, where $m = (v-2)/2$.
- Divide the resultant elements of A in i groups of size k_1 and two sets for k_2 such that the sum of each group should be divisible by v.
- Delete one element (any) from each group, resulting will be (i+2) sets of shifts to generate required MCGNDs in blocks of two different sizes.

Example 2.1. Following is MCGND generated from S_1 = [4,6,11,17], S_2 = [7,8], S_3 = [9,10] for v = 20, k_1 = 5 and k_2 = 3.

Take v blocks for S_1. Allocate 0, 1, …, v-1, treatments to the first unit for each block. Add 4 (mod v) to each element of first units to get the second unit elements. Similarly add 6 (mod 20) to each element of the second units to get the third unit elements, and so on, see table 1.

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Table 1: Blocks generated from S_1 = [4,6,11,17]

Blocks																			
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3
10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	0
18	19	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17

Take 20 more blocks for S_2 = [7,8] and generate blocks in the similar way as of S_1, see Table 2.

Table 2: Blocks generated from S_2 = [7,8]

Blocks																			
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
7	8	9	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6
15	16	17	18	19	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

Take 20 more blocks for S_3 = [9,10] and generate blocks in the similar way as of S_1, see Table 3.

Table 3: Blocks generated from S_3 = [9,10]

Blocks																			
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
9	10	11	12	13	14	15	16	17	18	19	0	1	2	3	4	5	6	7	8
19	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18

Table 1, Table 2 and Table 3 jointly present the MCGND for v = 20, k_1 = 5 and k_2 = 3.

3. MCGNDs in Blocks of Two Different Sizes in Which 3*v*/2 Unordered Pairs of Distinct Treatments Appear Twice as Neighbors.

In this Section, MCGNDs are constructed in blocks of two different sizes in which $3v/2$ unordered pairs of distinct treatments appear twice as neighbors while all others appear once. These designs are constructed from i sets of shifts for k_1 and two for k_2.

3.1 MCGNDs in blocks of two different sizes for m (mod 4) ≡ 1

Theorem 3.1.1:

If m (mod 4) ≡ 1 then (i+2) sets of shifts derived from A = [1, 2,…, (m-5)/4, (m+3)/4, (m+7)/4,…, m, (m+1), m, (7m+9)/4] provide MCGNDs in blocks of two different sizes for $v = 2ik_1+4k_2-2$ using i sets for k_1 and two for k_2, where $m = (v-2)/2$. In these designs $3v/2$ unordered pairs of distinct treatments appear twice as neighbors while all others appear once.

Generator 3.1.1.1

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.1.1 for $v = 2ik_1+10$, k_1 (mod 4) ≡ 1, k_2 = 3, i (mod 4) ≡ 1, m (mod 4) ≡ 1 and $m = (v-2)/2$.

Example 3.1.1.1

MCGND is constructed from the following sets for v = 20, k_1 = 5, k_2 = 3.

S_1 = [4,6,11,17], S_2 = [7,8], S_3 = [9,10]

Generator 3.1.1.2.

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.1.1 for $v = 2ik_1+10$, k_1 (mod 4) ≡ 3, k_2 = 3, i (mod 4) ≡ 3, m (mod 4) ≡ 1 and $m = (v-2)/2$.

Example 3.1.1.2.

MCGND is constructed from the following sets for v = 52, k_1 = 7, k_2 = 3.

S_1 = [3,4,5,18,27,45], S_2 = [11,12,13,14,22,23],
S_3 = [10,15,16,17,21,19], S_4 = [20,24], S_5 = [25,26]

Generator 3.1.1.3.

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.1.1 for $v = 2ik_1+14$, k_1 (mod 4) ≡ 1, k_2 = 4, i (mod 4) ≡ 3, m (mod 4) ≡ 1 and $m = (v-2)/2$.

Example 3.1.1.3.

MCGND is constructed from the following sets for v = 44, k_1 = 5, k_2 = 4.

S_1 = [19,20,22,23], S_2 = [7,8,9,15], S_3 = [12,13,14,38],
S_4 = [3,18,21], S_5 = [10,16,17]

Generator 3.1.1.4.

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.1.1 for $v = 2ik_1+10$, k_1 (mod 4) ≡ 3, k_2 = 4, i (mod 4) ≡ 1, m (mod 4) ≡ 1 and $m = (v-2)/2$.

Example 3.1.1.4.

MCGND is constructed from the following sets for v = 28, k_1 = 7, k_2 = 4.

S_1 = [7,8,10,14,15,24], S_2 = [5,9,11], S_3 = [2,12,13]

Generator 3.1.1.5.

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.1.1 for $v = 2ik_1+18$, k_1 (mod 4) ≡ 1, $k_1 > 5$, $k_2 = 5$, *i* (mod 4) ≡ 1, *m* (mod 4) ≡ 1 and $m = (v\text{-}2)/2$.

Example 3.1.1.5.

MCGND is constructed from the following sets of shifts for v = 36, k_1 = 9 and k_2 = 5.

S_1 = [10,12,13,16,17,18,19,31], S2 = [6,7,9,11],

S_3 = [2,4,14,15]

Generator 3.1.1.6.

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.1.1 for $v = 2ik_1+18$, k_1 (mod 4) ≡ 3, $k_2 = 5$, *i* (mod 4) ≡ 3, *m* (mod 4) ≡ 1 and $m = (v\text{-}2)/2$.

Example 3.1.1.6.

MCGND is constructed from the following sets for v = 60, k_1 = 7, k_2 = 5.

S_1 = [2,3,4,7,13,30], S_2 = [9,10,11,12,20,52],
S_3 = [15,16,17,18,19,21], S_4 = [23,24,25,26],
S_5 = [27,31,28,29]

3.2 MCGNDs in blocks of two different sizes for m (mod 4) ≡ 2

Theorem 3.2.1:

If *m* (mod 4) ≡ 2 then (*i*+2) sets of shifts derived from A = [1, 2, …, *m*/2, (*m*+4)/2,…, *m*, (*m*+1), (*m*+2), (3*m*+2)/2] provide MCGNDs in blocks of two different sizes for $v = 2ik_1+4k_2\text{-}2$ using *i* sets for k_1 and two for k_2, where $m = (v\text{-}2)/2$. In these designs 3*v*/2 unordered pairs of distinct treatments appear twice as neighbors while all others appear once.

Generator 3.2.1.1

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.2.1 for $v = 2ik_1+10$, $k_1 = 4l+2$, $k_2 = 3$, *i* odd, *l* integer, *m* (mod 4) ≡ 2 and $m = (v\text{-}2)/2$.

Example 3.2.1.1

MCGND is constructed from the following sets for v = 22, k_1 = 6, k_2 = 3.

S_1 = [3,4,7,12,16], S_2 = [8,9], S_3 = [10,11]

Generator 3.2.1.2

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.1.2 for $v = 2ik_1+10$, k_1 (odd) > 3, $k_2 = 3$, *i* (mod 4) ≡ 2, *m* (mod 4) ≡ 2 and $m = (v\text{-}2)/2$.

Example 3.2.1.2

MCGND is constructed from the following sets for v = 30, k_1 = 5, k_2 = 3.

S_1 = [3,4,5,16], S_2 = [9,10,12,22], S_3 = [11,13], S_4 = [14,15]

Generator 3.2.1.3

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.2.1 for $v = 2ik_1+14$, $k_1 = 4l$, $k_2 = 4$, *l* (integer) >1, *i* integer, *m* (mod 4) ≡ 2 and $m = (v\text{-}2)/2$.

Example 3.2.1.3

MCGND is constructed from the following sets for v = 30, k_1 = 8, k_2 = 4.

S_1 = [3,4,5,16,10,14,7], S_2 = [12,15,22], S_3 = [6,9,13]

Generator 3.2.1.4

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.2.1 for $v = 2ik_1+14$, $k_1 = 4l+2$, $k_2 = 4$, *i* even, *l* integer, *m* (mod 4) ≡ 2 and $m = (v\text{-}2)/2$.

Example 3.2.1.4

MCGND is constructed from the following sets for v = 38, k_1 = 6, k_2 = 4.

S_1 = [6,11,16,19,20], S_2 = [7,8,13,15,28], S_3 = [9,12,14],
S_4 = [2,17,18]

Generator 3.2.1.5

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.2.1 for $v = 2ik_1+14$, k_1 (odd) > 3, $k_2 = 4$, *i* (mod 4) ≡ 0, *m* (mod 4) ≡ 2 and $m = (v\text{-}2)/2$.

Example 3.2.1.5

MCGND is constructed from the following sets of shifts for v = 54, k_1 = 5 and k_2 = 4.

S_1 = [4,5,15,28], S_2 = [7,9,10,22], S_3 = [13,16,27,40],
S_4 = [20,21,23,25],
S_5 = [11,17,18], S_6 = [3,24,26]

Generator 3.2.1.6

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.2.1 for $v = 2ik_1+18$, $k_1 = 4l+2$, $k_2 = 5$, *i* odd, *l* integer, *m* (mod 4) ≡ 2 and $m = (v\text{-}2)/2$.

Example 3.2.1.6

MCGND is constructed from the following sets for v = 30, k_1 = 6, k_2 = 5.

S_1 = [2,3,4,5,15], S_2 = [9,10,12,22], S_3 = [16,11,14,13]

Generator 3.2.1.7

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.2.1 for $v = 2ik_1+20$, k_1 (odd) > 3, k_2=5, *i* (mod 4) ≡ 2, *m* (mod 4) ≡ 2 and $m = (v\text{-}2)/2$.

Example 3.2.1.7

MCGND is constructed from the following sets for v = 40, k_1 = 5, k_2 = 5.

S_1 = [4,5,11,15], S_2 = [8,13,20,28], S_3 = [2,9,12,14],
S_4 = [16,17,18,19]

3.3 MCGNDs in blocks of two different sizes for m (mod 4) ≡ 3

Theorem 3.3.1:

If m (mod 4) ≡ 3 then (i+2) sets of shifts derived from A = [1, 2,…, (3m-5)/4, (3m+3)/4, (3m+7)/4,…, m-1, m, m+1, m-1, (5m+9)/4] provide MCGNDs in blocks of two different sizes for $v = 2ik_1+4k_2-2$ using i sets for k_1 and two for k_2, where $m = (v-2)/2$. In these designs 3v/2 unordered pairs of distinct treatments appear twice as neighbors while all others appear once.

Generator 3.3.1.1

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.3.1 for $v = 2ik_1+10$, k_1 (mod 4) ≡ 1, k_2 = 3, i (mod 4) ≡ 3, m (mod 4) ≡ 3 and $m = (v-2)/2$.

Example 3.3.1.1

MCGND is constructed from the following sets for v = 40, $k_1 = 5$, $k_2 = 3$.

S_1 = [4,5,10,18], S_2 = [6,7,9,16],
S_3 = [12,13,18,26], S_4 = [15,17], S_5 = [19,20]

Generator 3.3.1.2

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.3.1 for $v = 2ik_1+10$, k_1 (mod 4) ≡ 3, k_2 = 3, i (mod 4) ≡1, m (mod 4) ≡3 and $m = (v-2)/2$.

Example 3.3.1.2

MCGND is constructed from the following sets for v = 24, $k_1 = 7$, $k_2 = 3$.

S_1 = [2,4,6,7,12,16], S_2 = [9,10], S_3 = [10,11]

Generator 3.3.1.3

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.3.1 for $v = 2ik_1+14$, k_1 (mod 4) ≡ 1

k_2 = 4, i(mod 4) ≡ 1, m (mod 4) ≡ 3 and $m = (v-2)/2$.

Example 3.3.1.3

MCGND is constructed from the following sets for v = 24, $k_1 = 5$, $k_2 = 4$.

S_1 = [3,4,5,11], S_2 = [6,7,9], S_3 = [10,12,16]

Generator 3.3.1.4

MCGNDs can be constructed from i sets of shifts for k1 and two for k2 using theorem 3.3.1 for $v = 2ik_1+10$, k_1 (mod 4) ≡ 3, k_2 = 4, i (mod 4) ≡ 3, m (mod 4) ≡ 3 and $m = (v-2)/2$.

Example 3.3.1.4

MCGND is constructed from the following sets for v = 56, $k_1 = 7$, $k_2 = 4$.

S_1=[3,4,5,6,9,28], S_2=[11,12,13,14,25,27],
S_3=[19,22,23,24,26,36], S_4 = [15,16,17], S_5 = [7,21,26]

Generator 3.3.1.5

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.3.1 for $v = 2ik_1+18$, k_1 (mod 4) ≡ 1, k_2 =5, i (mod 4) ≡ 3, m (mod 4)≡3 and $m = (v-2)/2$.

Example 3.3.1.5

MCGND is constructed from the following sets for v = 72, $k_1 = 9$, $k_2 = 5$.

S_1 = [10,11,12,14,15,19,20,21,22],
S_2 = [8,16,17,18,27,30,31,34,35],
S_3 = [6,9,13,23,24,25,34,36,46], S_4 = [3,5,7,28,29],
S_5 = [1,2,4,32,33]

Generator 3.3.1.6

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.3.1 for $v = 2ik_1+18$, k_1 (mod 4) ≡ 3, k_2 = 5, i (mod 4) ≡ 1, m (mod 4) ≡ 3 and $m = (v-2)/2$.

Example 3.3.1.6

MCGND is constructed from the following sets for v = 32, $k_1 = 7$, $k_2 = 5$.

S_1 = [2,3,5,6,7,8], S_2 = [12,13,14,21],S_3 = [10,14,15,16]

3.4 MCGNDs in blocks of two different sizes for m (mod 4) ≡ 0

Theorem 3.4.1:

If m (mod 4) ≡ 0 then (i+2) sets of shifts derived from A = [1, 2,…, m-1, m-1, m+1, m+2] provide MCGNDs in blocks of two different sizes for $v = 2ik_1+4k_2-2$ using i sets for k_1 and two for k_2, where $m = (v-2)/2$. In these designs 3v/2 unordered pairs of distinct treatments appear twice as neighbors while all others appear once.

Generator 3.4.1.1

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.4.1 for $v = 2ik_1+10$, $k_1 = 4l+2$, k_2 = 3, i even, l integer, m (mod 4) ≡ 0 and $m = (v-2)/2$.

Example 3.4.1.1

MCGND is constructed from the following sets for v = 36, $k_1 = 6$, $k_2 = 3$.

S_1=[2,3,5,6,17], S_2=[9,10,11,12,18], S_3=[13,14],S_4 = [15,15]

Generator 3.4.1.2

MCGNDs can be constructed from i sets of shifts for k_1 and two for k_2 using theorem 3.4.1 for $v = 2ik_1+10$, k_1 (odd) > 3, k_2 = 3, i (mod 4) ≡ 0, m (mod 4) ≡ 0 and $m = (v-2)/2$.

Example 3.4.1.2

MCGND is constructed from the following sets for v = 50, $k_1 = 5$, $k_2 = 3$.

S_1 = [3,4,16,26], S_2 = [7,8,10,19], S_3 = [9,11,12,13], S_4 = [18,20,22,23], S_5 = [15,21], S_6 =[23,25]

Generator 3.4.1.3

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.4.1 for $v = 2ik_1+14$, $k_1 = 4l+2$, $k_2 = 4$, *i* odd, *l* integer, $m \pmod 4 \equiv 0$ and $m = (v-2)/2$.

Example 3.4.1.3

MCGND is constructed from the following sets for v = 26, k_1 = 6, k_2 = 4.

S_1 = [6,7,10,11,14], S_2 = [3,9,13], S_3 = [5,8,11]

Generator 3.4.1.4

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.4.1 for $v = 2ik_1+14$, k_1 (odd) > 3, $k_2 = 4$, $i \pmod 4 \equiv 2$, $m \pmod 4 \equiv 0$ and $m = (v-2)/2$.

Example 3.4.1.4

MCGND is constructed from the following sets of shifts for v = 34, k_1 = 5 and k_2 = 4.

S_1=[3,5,10,15], S_2=[12,14,15,18], S3=[7,8,13], S_4=[4,11,17]

Generator 3.4.1.5

MCGNDs can be constructed from the *i* sets of shifts for k_1 and two for k_2 using theorem 3.4.1 for $v = 2ik_1+18$, $k_1 = 4l+2$, $k_2 = 5$, *i* even, *l* integer, $m \pmod 4 \equiv 0$ and $m = (v-2)/2$.

Example 3.4.1.5

MCGND is constructed from the following sets for v = 42,

k_1 = 6, k_2 = 5.

S_1 = [3,5,6,8,19], S_2 = [12,13,15,16,17], S_3 = [7,9,10,14], S_4 = [18,19,21,22]

Generator 3.4.1.6

MCGNDs can be constructed from *i* sets of shifts for k_1 and two for k_2 using theorem 3.4.1 for $v = 2ik_1+18$, k_1 (odd) > 3, $k_2 = 5$, $i \pmod 4 \equiv 0$, $m \pmod 4 \equiv 0$ and $m = (v-2)/2$.

Example 3.4.1.6

MCGND is constructed from the following sets of shifts for v = 58, k_1 = 5 and k_2 = 5.

S_1 = [22,23,25,26], S_2 = [9,10,14,24], S_3 = [11,12,13,15], S_4 = [4,16,17,19], S_5 = [6,8,18,21], S_6 = [27,27,29,30]

4. Catalogues of MCGNDs in Blocks of Two Different Sizes.

In this Section, catalogues of MCGNDs are presented in blocks of two different sizes in which $3v/2$ unordered pairs of treatments appear twice while remaining ones appear once. Catalogues provide the readymade solution to the experimenters and researchers.

4.1 Catalogues of MCGNDs for m (mod 4) ≡ 0

Table 4: MCGNDs for $v = 2ik_1+10$, $k_1 = 4l+2$, $k_2 = 3$, *i* even, $m \pmod 4 \equiv 0$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
34	6	3	[1,2,3,5,6,17]+[8,9,10,11,12,18]+[7,13,14]+[4,15,15]
58	6	3	[1,3,4,5,15,30]+[7,8,9,10,11,13]+[14,16,17,19,24,26]+[12,18,20,21,22,23]+ [6,25,27]+[2,27,29]
82	6	3	[1,5,6,22,23,25]+[7,8,9,10,11,37]+[16,17,18,35,36,42]+[3,4,15,19,20,21]+[24,26,27,28,29,30]+[13,14,32,33,34,38]+ [12,31,39]+[2,39,41]
50	10	3	[1,3,4,6,7,8,9,15,21,26]+[10,11,12,13,16,14,17,18,19,20]+[5,22,23]+[2,23,25]
90	10	3	[1,2,3,4,5,6,7,8,9,45]+[11,12,13,14,15,17,18,19,20,41]+[22,23,24,25,26,27,28,30,33,32]+[21,29,35,36,37,38,39,40,42,43]+ [16,31,43]+[10,34,46]
66	14	3	[1,2,3,4,5,7,8,9,10,11,13,12,14,33]+[16,18,19,20,21,22,24,23,25,27,26,28,30,31]+[15,17,34]+[6,29,31]
82	18	3	[1,3,4,6,7,8,9,10,11,12,13,14,15,16,17,21,37,42]+[18,19,20,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36]+[5,38,39]+[2,39,41]

Table 5: MCGNDs for $v = 2i\,k_1+ 10$, k_1 (odd) > 3, $k_2 = 3$, $i \pmod 4 \equiv 0$, $m \pmod 4 \equiv 0$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
50	5	3	[1,3,4,16,26]+[6,7,8,10,19]+[5,9,11,12,13]+[17,18,20,22,23]+[14,15,21]+[2,23,25]
90	5	3	[32,33,34,35,46]+[7,8,9,24,42]+[11,12,13,25,29]+[26,37,38,39,40]+[10,14,21,22,23]+[1,15,19,27,28]+[4,5,20,30,31]+ [3,16,17,18,36]+ [6,41,43]+[2,43,45]
66	7	3	[5,10,11,14,28,30,34]+[1,3,7,8,9,12,26]+[15,16,17,27,20,18,19]+[4,13,21,22,24,23,25]+[6,29,31]+[2,31,33]
82	9	3	[1,2,3,4,6,7,8,9,42]+[10,13,14,15,16,17,18,20,41]+[19,22,23,25,26,27,31,36,37]+[12,21,24,28,29,30,33,34,35]+ [11,32,39]+[5,38,39]
98	11	3	[3,4,6,7,8,9,10,21,41,37,50]+[11,12,13,14,16,17,18,19,20,32,24]+[15,22,23,25,26,27,28,29,30,38,31]+ [1,33,34,35,36,39,40,42,43,44,45]+[5,46,47]+[2,47,49]

Table 6: MCGNDs for $v = 2ik_1+14$, $k_1 = 4l+2$, $k_2 = 4$, *i* odd, $m \pmod 4 \equiv 0$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
26	6	4	[4,6,7,10,11,14]+[1,3,9,13]+[2,5,8,11]
50	6	4	[2,4,5,6,7,26]+[10,11,13,19,22,25]+[14,15,16,17,18,20]+[8,9,12,21]+[1,3,23,23]
74	6	4	[8,15,28,29,30,38]+[2,3,9,11,12,37]+[17,18,19,27,33,34]+[21,22,23,24,26,32]+[4,5,6,14,20,25]+[10,13,16,35]+[1,7,31,35]

98	6	4	[3,4,5,6,30,50]+[7,9,10,11,12,49]+[13,14,15,16,17,23]+[20,21,24,40,45,46]+[27,28,29,31,34,47]+[25,26,32,33,36,44]+ [19,22,37,38,39,41]+ [2,18,35,43]+[1,8,42,47]
34	10	4	[3,4,7,8,9,12,10,14,17,18]+[2,6,11,15]+[1,5,13,15]
74	10	4	[1,2,3,4,5,6,7,8,9,29]+[18,19,20,30,32,33,34,35,37,38]+[10,13,21,22,23,24,25,26,27,31]+[14,15,17,28]+[11,12,16,35]
42	14	4	[3,4,5,7,9,10,12,13,11,16,18,19,19,22]+[2,8,15,17]+[1,6,14,21]
98	14	4	[2,3,4,6,7,8,9,10,11,12,13,14,47,50]+[17,18,19,21,22,23,24,26,27,28,32,44,45,46]+[25,29,30,31,34,35,33,36,37,38,39,40,41,42]+ [15,16,20,47]+[1,5,43,49]
50	18	4	[1,4,5,6,7,9,11,13,12,15,16,18,17,20,25,23,22,26]+[2,8,19,21]+[3,10,14,23]

Table 7: MCGNDs for $v = 2i\,k_1 + 14$, k_1 (odd) > 3, $k_2 = 4$, $i \pmod 4 \equiv 2$, $m \pmod 4 \equiv 0$ and $v \leq 100$

v	**k$_1$**	**k$_2$**	**Sets of Shifts**
34	5	4	[1,3,5,10,15]+[9,12,14,15,18]+[6,7,8,13]+[2,4,11,17]
74	5	4	[2,3,20,24,25]+[8,9,10,12,35]+[11,13,14,15,21]+[4,16,17,18,19]+[23,26,27,34,38]+[22,29,30,32,35]+[6,7,28,33]+[1,5,31,37]
42	7	4	[2,3,7,13,18,19,22]+[6,8,10,11,12,16,21]+[1,9,15,17]+[4,5,14,19]
98	7	4	[2,3,4,5,6,28,50]+[11,12,13,20,45,46,49]+[16,18,19,26,38,35,44]+[21,22,23,24,25,34,47]+[14,27,29,30,31,32,33]+ [7,15,17,36,39,40,42] [8,10,37,43]+[1,9,41,47]
50	9	4	[3,5,6,7,8,11,14,21,25]+[10,26,13,15,16,17,23,18,12]+[2,9,19,20]+[1,4,22,23]
58	11	4	[2,3,4,5,7,8,9,10,11,27,30]+[15,16,17,18,20,21,22,23,25,26,29]+[12,13,14,19]+[1,6,24,27]
66	13	4	[4,5,6,7,8,10,11,13,20,21,29,30,34]+[12,14,15,16,17,19,18,22,23,24,25,26,33]+[2,9,27,28]+[1,3,31,31]
74	15	4	[2,3,4,5,68,9,11,12,13,30,15,34,35,35]+[17,18,38,20,2122,23,25,24,26,27,28,29,33,19]+[1,10,31,32]+[7,14,16,37]
82	17	4	[2,3,4,5,6,7,9,10,11,13,15,14,16,19,38,35,39]+[20,21,22,23,24,25,26,27,28,29,30,31,33,36,41,34,42]+[1,12,32,37]+[8,17,18,39]
90	19	4	[2,5,6,7,8,9,10,12,13,15,17,18,19,22,27,39,40,45,46]+[11,14,20,21,23,24,25,26,28,30,31,32,34,33,35,36,37,42,38]+[4,16,29,41]+ [1,3,43,43]

Table 8: MCGNDs for $v = 2ik_1 + 18$, $k_1 = 4l + 2$, $k_2 = 5$, i even, $m \pmod 4 \equiv 0$ and $v \leq 100$

v	**k$_1$**	**k$_2$**	**Sets of Shifts**
42	6	5	[1,3,5,6,8,19]+[11,12,13,15,16,17]+[2,7,9,10,14]+[4,18,19,21,22]
66	6	5	[1,2,6,13,14,30]+[7,8,10,11,12,18]+[4,5,9,15,16,17]+ [19,20,21,23,27,22]+ [24,25,26,28,29]+[3,31,31,33,34]
90	6	5	[1,2,6,19,20,42]+[7,8,9,11,24,31]+[12,13,14,16,17,18]+[4,5,15,21,22,23]+[26,27,28,29,30,40]+[10,32,33,34,35,36]+ [25,37,38,39,41]+[3,43,43,45,46]
58	10	5	[1,2,3,4,5,6,7,8,9,13]+[10,11,12,14,15,16,19,20,27,30]+[21,22,23,24,26]+[17,18,25,27,29]
98	10	5	[1,2,3,4,5,6,8,9,10,50]+[12,13,14,15,16,17,18,19,33,39]+[21,23,24,25,27,26,29,38,40,41]+[11,20,28,30,31,32,34,35,36,37]+ [22,42,43,44,45]+[7,46,47,47,49]
74	14	5	[1,2,3,5,7,8,9,10,11,12,13,14,18,35]+[6,15,16,17,19,20,21,23,24,25,26,27,28,29]+[22,30,31,32,33]+[4,34,35,37,38]
90	18	5	[1,2,3,4,5,6,7,8,9,11,12,13,14,15,17,16,18,19]+[21,29,23,24,25,26,27,38,28,30,33,31,34,35,36,32,46,22]+ [20,37,39,41,43]+[10,40,42,43,45]

Table 9: MCGNDs for $v = 2i\,k_1 + 18$, k_1 (odd) > 3, $k_2 = 5$, $i \pmod 4 \equiv 0$, $m \pmod 4 \equiv 0$ and $v \leq 100$

v	**k$_1$**	**k$_2$**	**Sets of Shifts**
58	5	5	[20,22,23,25,26]+[1,9,10,14,24]+[7,11,12,13,15]+[2,4,16,17,19]+[5,6,8,18,21]+[3,27,27,29,30]
98	5	5	[34,36,38,41,47]+[5,10,12,31,40]+[2,9,25,30,32]+[16,17,18,19,28]+[7,15,23,24,29]+[11,14,20,26,27]+ [1,8,21,33,35]+[3,6,13,37,39]+[22,42,43,44,45]+[4,46,47,49,50]
74	7	5	[1,2,4,5,10,18,34]+[7,8,9,11,12,13,14]+[15,17,19,20,21,27,29]+[6,16,23,24,25,26,28]+[22,30,31,32,33]+[3,35,35,37,38]
90	9	5	[1,2,4,5,6,7,8,15,42]+[10,11,12,13,14,17,27,36,40]+[9,16,19,20,21,22,23,26,24]+[18,28,29,30,32,31,33,34,35]+[25,37,38,39,41]+[3,43,43,45,46]

4.2 Catalogues of MCGNDs for m (mod 4) ≡ 1

Table 10: MCGNDs for $v = 2ik_1 + 10$, $k_1 \pmod 4 \equiv 1$, $k_2 = 3$, $i \pmod 4 \equiv 1$, $m \pmod 4 \equiv 1$ and $v \leq 100$

v	**k$_1$**	**k$_2$**	**Sets of Shifts**
20	5	3	[2,4,6,11,17]+[5,7,8]+[1,9,10]
60	5	3	[2,3,5,19,31]+[6,14,20,28,52]+ [7,9,11,15,18]+[4,10,13,16,17]+ [22,23,24,25,26]+[12,21,27]+[1,29,30]
100	5	3	[2,3,4,40,51]+[31,36,42,43,48]+[10,14,15,22,39]+[16,17,19,20,28]+[21,25,33,34,87]+[8,18,23,24,27]+ [32,38,41,44,45]+[5,11,12,35,37]+ [6,9,26,29,30]+[7,46,47]+[1,49,50]
28	9	3	[2,3,5,6,8,12,9,15,24]+[7,10,11]+[1,13,14]
100	9	3	[2,3,4,5,6,7,8,14,51]+[9,12,16,19,23,41,46,47,87]+[15,17,18,20,21,22,24,25,38]+[28,29,30,31,32,33,34,35,48]+

			[10,11,26,39,40,42,43,44,45]+[27,36,37]+[1,49,50]
36	13	3	[2,3,4,7,8,9,10,11,13,19,12,31,15]+[6,14,16]+[1,17,18]
44	17	3	[2,3,4,5,8,9,10,11,12,13,14,15,16,17,20,23,38]+[7,18,19]+[1,21,22]

Table 11: MCGNDs for $v = 2i\ k_1+10$, $k_1 \pmod 4 \equiv 3$, $k_2 = 3$, $i \pmod 4 \equiv 3$, $m \pmod 4 \equiv 1$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
52	7	3	[2,3,4,5,18,27,45]+[9,11,12,13,14,22,23]+[6,10,15,16,17,21,19]+[8,20,24]+[1,25,26]
76	11	3	[1,2,3,4,5,6,7,8,11,66,39]+[14,15,16,17,18,19,20,21,24,28,36]+[12,22,23,25,26,27,31,32,33,35,38]+[13,29,34]+[9,30,37]
100	15	3	[1,2,3,4,5,6,7,8,9,10,11,12,14,21,87]+[17,18,19,20,22,23,24,25,26,27,28,29,30,51,41]+ [31,32,33,34,36,37,39,40,42,43,44,45,46,48,50]+[15,38,47]+[16,35,49]

Table 12: MCGNDs for $v = 2i\ k_1+14$, $k_1 \pmod 4 \equiv 1$, $k_2 = 4$, $i \pmod 4 \equiv 3$, $m \pmod 4 \equiv 1$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
44	5	4	[4,19,20,22,23]+[5,7,8,9,15]+[11,12,13,14,38]+[2,3,18,21]+[1,10,16,17]
84	5	4	[4,5,21,24,30]+[8,9,15,16,36]+[12,20,22,41,73]+[26,28,29,42,43]+[31,32,33,34,38]+[10,14,18,19,23]+[2,13,17,25,27]+ [1,7,37,39]+[3,6,35,40]
68	9	4	[1,2,7,8,29,30,35,59,33]+[10,11,12,13,14,15,17,18,26]+[16,19,20,21,22,23,24,27,32]+[4,5,28,31]+[3,6,25,34]
92	13	4	[1,2,3,4,5,6,7,8,9,11,13,35,80]+[17,18,20,21,22,23,25,26,27,41,40,43,45]+[28,29,30,31,32,33,34,36,37,39,46,38,47]+ [15,16,19,42]+[10,14,24,44]

Table 13: MCGNDs for $v = 2ik_1+14$, $k_1 \pmod 4 \equiv 3$, $k_2 = 4$, $i \pmod 4 \equiv 1$, $m \pmod 4 \equiv 1$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
28	7	4	[6,7,8,10,14,15,24]+[3,5,9,11]+[1,2,12,13]
84	7	4	[6,9,14,21,37,38,43]+[4,7,10,12,20,73,42]+[16,17,18,19,25,34,39]+[5,22,23,24,26,28,40]+[1,2,29,32,33,35,36]+[8,15,30,31]+ [3,13,27,41]
36	11	4	[4,6,7,8,9,13,15,18,14,19,31]+[3,10,11,12]+[1,2,16,17]
44	15	4	[3,4,5,7,8,12,13,14,15,18,19,20,23,38,21]+[2,9,16,17]+[1,10,11,22]
52	19	4	[4,5,8,9,10,11,12,13,14,15,16,17,18,19,20,23,45,26,27]+[3,6,21,22]+[1,2,24,25]

Table 14: MCGNDs for $v = 2ik_1+18$, $k_1 \pmod 4 \equiv 1$, $k_2 = 5$, $i \pmod 4 \equiv 1$, $m \pmod 4 \equiv 1$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
28	5	5	[1,2,3,8,14]+[6,7,9,10,24]+[5,11,13,12,15]
68	5	5	[3,4,7,20,34]+[8,16,24,29,59]+[1,12,14,18,23]+[6,11,15,17,19]+[2,10,13,21,22]+[25,26,27,28,30]+[5,31,33,32,35]
36	9	5	[1,2,3,4,6,7,8,10,31]+[11,12,17,13,19]+[9,14,15,16,18]
44	13	5	[1,2,3,4,5,7,8,9,12,11,19,38,13]+[15,16,17,18,22]+[10,14,23,20,21]
52	17	5	[1,2,3,4,5,6,8,9,11,12,13,14,15,17,16,27,45]+[18,19,20,21,26]+[10,22,23,24,25]

4.3 Catalogues of MCGNDs for m (mod 4) ≡ 2

Table 15: MCGNDs for $v = 2ik_1+10$, $k_1 = 4l+2$, $k_2 = 3$, i odd, $m \pmod 4 \equiv 2$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
22	6	3	[2,3,4,7,12,16]+[5,8,9]+[1,10,11]
46	6	3	[1,2,3,4,16,20]+[7,8,9,10,24,34]+[5,14,15,18,17,23]+ [6,19,21]+[11,13,22]
70	6	3	[6,20,21,28,29,36]+[4,7,8,11,15,25]+[13,14,16,19,26,52]+[10,22,23,24,30,31]+[2,3,9,12,17,27]+[5,32,33]+[1,34,35]
94	6	3	[2,3,5,6,30,48]+[9,10,12,42,45,70]+[8,13,16,17,19,21]+[26,27,28,29,38,40]+[4,11,14,18,22,25]+[20,31,32,34,35,36]+ [15,23,33,37,39,41]+[7,43,44]+[1,46,47]
30	10	3	[2,12,22,5,7,10,3,4,16,9]+[6,11,13]+[1,14,15]
70	10	3	[1,2,3,4,5,6,7,8,9,25]+[11,12,14,15,16,17,19,20,34,52]+ [22,23,24,26,27,28,30,32,33,35]+ [10,29,31]+[13,21,36]
38	14	3	[2,3,4,6,7,8,9,11,12,13,14,15,20,28]+[5,16,17]+[1,18,19]
94	14	3	[2,3,4,5,6,7,8,10,11,12,13,14,45,48]+[16,17,18,19,20,21,22,23,25,26,27,28,44,70]+[15,29,30,31,32,33,34,35,36,37,38,39,40,41]+ [9,42,43]+[1,46,47]
46	18	3	[2,3,4,5,7,8,9,11,10,13,14,16,15,18,20,17,24,34]+[6,19,21]+[1,22,23]

Table 16: MCGNDs for $v = 2ik_1+ 10$, k_1 (odd) > 3, $k_2 = 3$, $i \pmod 4 \equiv 2$, $m \pmod 4 \equiv 2$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
30	5	3	[2,3,4,5,16]+[7,9,10,12,22]+[6,11,13]+[1,14,15]
70	5	3	[2,3,4,25,36]+[7,8,9,14,32]+[10,12,13,15,20]+[19,22,23,24,52]+[5,11,16,17,21]+[26,27,28,29,30]+[6,31,33]+[1,34,35]

38	7	3	[1,2,3,4,5,6,17]+[9,12,13,14,20,28,18]+[7,15,16]+[8,11,19]
94	7	3	[2,3,4,5,7,25,48]+[8,9,10,11,12,14,30]+[17,20,21,23,28,35,44]+ [6,18,19,22,27,70,26]+[13,16,29,31,32,33,34]+ [37,38,39,40,41,42,45]+[15,36,43]+[1,46,47]
46	9	3	[1,2,3,4,5,6,7,8,10]+[13,15,17,18,20,21,24,34,22]+[11,16,19]+[9,14,23]
54	11	3	[2,3,4,5,6,7,8,11,24,10,28]+[12,13,15,16,17,19,20,18,21,25,40]+[9,22,23]+[1,26,27]
62	13	3	[2,3,4,5,6,7,8,9,10,11,13,14,32]+[15,17,18,19,20,21,23,24,25,26,27,29,46]+[12,22,28]+[1,30,31]
70	15	3	[1,2,3,4,6,7,8,9,10,11,13,14,12,15,25]+[17,19,20,21,23,24,26,27,28,29,36,33,52,34,31]+[16,22,32]+[5,30,35]
78	17	3	[1,2,3,4,5,6,7,8,9,10,15,19,11,17,14,13,12]+[21,23,24,26,27,28,29,30,31,32,33,34,35,36,39,40,58]+[16,25,37]+[18,22,38]
86	19	3	[2,3,4,5,6,7,8,9,10,11,12,13,15,17,16,18,19,39,44]+[20,21,23,24,25,26,27,28,29,30,33,32,34,35,36,37,40,64,38]+[14,31,41]+[1,42,43]

Table 17: MCGNDs for $v = 2ik_1+14$, $k_1 = 4l$, $k_2 = 4$, i integer, $m \pmod 4 \equiv 2$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
30	8	4	[1,3,4,5,16,10,14,7]+[11,12,15,22]+[2,6,9,13]
46	8	4	[1,2,4,5,18,22,17,23]+[9,11,14,15,16,19,20,34]+[3,6,13,24]+[7,8,10,21]
62	8	4	[1,2,3,5,6,7,14,24]+[8,9,10,11,13,15,28,30]+[18,19,20,21,22,23,31,32]+ [25,26,27,46]+[4,12,17,29]
78	8	4	[1,2,3,4,5,6,17,40]+[10,11,13,14,15,16,38,39]+[7,12,18,19,21,22,23,34]+[25,31,27,28,36,29,32,26]+[30,33,35,58]+[8,9,24,37]
94	8	4	[1,3,4,5,6,7,20,48]+[8,9,10,11,12,13,15,16]+[14,21,22,23,42,43,47,70]+[26,27,28,31,39,41,44,46]+[29,33,34,35,36,37,38,40]+ [18,19,25,32]+[2,17,30,45]
38	12	4	[5,6,7,8,9,11,12,13,14,19,20,28]+[3,4,15,16]+[1,2,17,18]
62	12	4	[3,4,7,8,9,10,12,15,27,28,31,32]+[11,13,14,17,19,18,20,21,22,23,24,46]+[5,6,25,26]+[1,2,29,30]
86	12	4	[1,3,4,5,6,7,10,11,38,40,27,20]+[12,13,14,15,16,17,18,19,21,23,26,64]+[29,30,31,32,33,34,35,36,41,42,43,44]+ [2,8,37,39]+[9,24,25,28]
46	16	4	[1,3,5,6,7,8,9,10,11,13,14,15,22,16,24,20]+[18,19,21,34]+[2,4,17,23]
78	16	4	[1,2,3,5,7,8,9,11,12,13,15,16,28,30,36,38]+[18,19,21,22,23,24,25,26,29,31,39,40,34,58,27,32]+[4,6,33,35]+[10,14,17,37]
54	20	4	[1,2,4,5,6,7,8,9,10,11,15,16,18,17,20,19,22,25,27,28]+[21,23,24,40]+[3,12,13,26]
94	20	4	[1,3,4,5,6,7,8,9,10,11,16,13,48,15,17,12,19,14,46,18]+[26,27,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,70]+ [21,22,23,28]+[2,20,25,47]

Table 18: MCGNDs for $v = 2ik_1+14$, $k_1 = 4l+2$, $k_2 = 4$, i even, $m \pmod 4 \equiv 2$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
38	6	4	[4,6,11,16,19,20]+[5,7,8,13,15,28]+[3,9,12,14]+[1,2,17,18]
62	6	4	[1,3,4,5,17,32]+[6,12,20,27,28,31]+[10,11,13,14,30,46]+[9,15,21,24,26,29]+[7,8,22,25]+[2,18,19,23]
86	6	4	[16,17,18,37,40,44]+[7,8,9,10,11,41]+[1,2,3,6,35,39]+[5,19,20,30,34,64]+[23,24,26,28,29,42]+[15,25,31,32,33,36]+ [13,14,21,38]+[4,12,27,43]
54	10	4	[3,4,5,6,7,8,9,11,27,28]+[10,12,13,15,16,17,19,20,18,22]+[21,23,24,40]+[1,2,25,26]
94	10	4	[1,2,3,5,7,8,32,45,39,46]+[12,13,14,15,16,17,18,19,30,34]+[11,21,22,23,25,26,27,28,29,70]+[20,33,35,36,37,38,43,42,44,48]+ [4,9,40,41]+[6,10,31,47]
70	14	4	[2,4,5,6,7,8,9,11,12,13,31,36,14,52]+[3,20,21,22,23,24,25,26,27,28,30,32,34,35]+[10,15,16,29]+[1,17,19,33]
86	18	4	[2,3,4,6,7,8,9,11,12,13,14,15,16,17,19,23,39,40]+[20,24,25,26,27,28,29,30,31,32,33,34,35,36,41,44,43,64]+[1,10,37,38]+[5,18,21,42]

Table 19: MCGNDs for $v = 2ik_1+14$, k_1 (odd) > 3, $k_2 = 4$, $i \pmod 4 \equiv 0$, $m \pmod 4 \equiv 2$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
54	5	4	[2,4,5,15,28]+[6,7,9,10,22]+[12,13,16,27,40]+[19,20,21,23,25]+[8,11,17,18]+[1,3,24,26]
94	5	4	[3,4,9,30,48]+[5,7,10,27,45]+[8,11,14,20,41]+[15,16,18,22,23]+[17,21,34,46,70]+[2,13,25,26,28]+[29,33,37,42,47]+ [35,36,38,39,40]+[6,12,32,44]+[1,19,31,43]
70	7	4	[2,3,4,5,6,14,36]+[7,8,9,10,11,12,13]+[15,16,17,19,20,21,32]+[25,26,27,30,33,34,35]+[28,29,31,52]+[1,22,23,24]
86	9	4	[4,5,6,7,8,29,30,41,42]+[9,11,12,14,15,16,17,34,44]+[19,20,21,23,25,26,27,64,33]+[2,13,24,31,35,36,38,39,40]+[3,18,28,37]+ [1,10,32,43]

Table 20: MCGNDs for $v = 2ik_1+18$, $k_1 = 4l+2$, $k_2 = 5$, i odd, $m \pmod 4 \equiv 2$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
30	6	5	[1,2,3,4,5,15]+[7,9,10,12,22]+[6,16,11,14,13]
54	6	5	[1,2,3,7,16,25]+[4,5,9,11,12,13]+[8,10,15,17,18,40]+ [19,20,21,22,26]+[6,23,24,27,28]
78	6	5	[2,3,4,5,24,40]+[7,9,11,12,16,23]+[8,10,13,14,15,18]+[1,17,19,30,31,58]+[21,25,26,28,27,29]+ [22,32,33,34,35]+[6,36,37,38,39]
38	10	5	[1,2,3,4,5,6,7,8,28,12]+[11,20,13,14,18]+[9,15,16,17,19]

78	10	5	[1,2,4,5,6,7,8,9,10,26]+[12,13,14,15,16,17,18,31,40,58]+ [3,21,22,23,24,25,27,28,32,29]+[19,30,33,35,39]+[11,34,36,37,38]
46	14	5	[1,2,3,4,5,7,8,9,11,10,13,14,17,34]+[15,16,18,19,24]+[6,20,21,22,23]
54	18	5	[1,2,3,4,5,7,8,9,10,12,11,13,15,16,17,18,25,40]+[19,20,21,22,26]+[6,23,24,27,28]

Table 21: MCGNDs for $v = 2i\,k_1 + 18$, k_1 (odd) > 3, $k_2 = 5$, $i \pmod 4 \equiv 2$, $m \pmod 4 \equiv 2$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
38	5	5	[3,4,5,11,15]+[7,8,13,20,28]+[1,2,9,12,14]+[6,16,17,18,19]
78	5	5	[2,3,4,30,39]+[7,8,9,19,35]+[11,13,15,16,23]+[22,24,25,27,58]+[10,12,17,18,21]+[1,6,14,28,29]+[26,31,32,33,34]+[5,36,38,37,40]
46	7	5	[1,2,3,4,5,7,24]+[9,10,11,13,14,16,19]+[8,15,17,18,34]+[6,20,21,22,23]
54	9	5	[1,2,3,4,5,6,7,8,18]+[10,11,12,13,15,16,17,28,40]+[19,20,21,22,26]+[9,23,24,25,27]
62	11	5	[1,4,5,6,7,9,10,11,20,23,28]+[3,8,12,13,14,17,15,18,19,21,46]+[22,24,25,26,27]+[2,29,30,31,32]
70	13	5	[1,2,3,4,5,8,9,10,11,12,13,26,36]+[7,14,15,16,17,19,20,21,22,23,24,52,30]+[25,27,28,29,31]+[6,32,33,34,35]
78	15	5	[1,3,4,5,6,7,8,9,10,11,12,13,14,17,36]+[15,16,18,19,21,23,24,25,27,28,26,29,30,31,58]+[22,32,33,34,35]+[2,37,38,39,40]
86	17	5	[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,17,35]+[19,20,21,23,24,25,26,27,28,29,31,32,33,34,36,44,64]+[16,37,38,39,42]+[18,30,40,41,43]
94	19	5	[1,2,4,5,7,8,9,10,11,12,13,14,15,16,18,19,34,48,36]+[3,17,20,21,23,25,26,27,28,29,31,30,32,33,35,38,37,39,70]+[22,40,41,42,43]+ [6,44,45,46,47]

4.4 *Catalogues of MCGNDs for m (mod 4) ≡ 3*

Table 22: MCGNDs for $v = 2i\,k_1 + 10$, $k_1 \pmod 4 \equiv 1$, $k_2 = 3$, $i \pmod 4 \equiv 1$, $m \pmod 4 \equiv 3$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
40	5	3	[3,4,5,10,18]+[2,6,7,9,16]+[11,12,13,18,26]+[8,15,17]+[1,19,20]
80	5	3	[3,11,12,16,38]+[7,8,10,18,37]+[5,13,14,20,28]+[4,15,17,19,25]+[2,9,22,23,24]+[21,26,27,35,51]+[30,31,32,33,34]+[6,36,38]+ [1,39,40]
64	9	3	[1,2,3,4,5,6,7,8,28]+[10,11,12,13,14,15,16,17,20]+[21,22,24,26,29,30,31,32,41]+[9,25,30]+[18,19,27]
88	13	3	[2,3,4,6,7,8,9,10,11,12,22,40,42]+[2,3,4,6,7,8,9,10,11,12,22,40,42]+[13,14,15,16,17,18,19,21,23,24,25,30,29]+[20,26,27]+[5,41,42]

Table 23: MCGNDs for $v = 2i\,k_1 + 10$, $k_1 \pmod 4 \equiv 3$, $k_2 = 3$, $i \pmod 4 \equiv 1$, $m \pmod 4 \equiv 3$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
24	7	3	[1,2,4,6,7,12,16]+[5,9,10]+[3,10,11]
80	7	3	[2,3,4,5,6,22,38]+[8,9,10,11,12,13,17]+[14,16,18,19,20,35,38]+[7,21,23,24,26,28,31]+[25,27,30,34,36,37,51]+[15,32,33]+[1,39,40]
32	11	3	[2,3,4,6,7,8,9,10,12,14,21]+[5,13,14]+[1,15,16]
40	15	3	[2,3,4,6,7,8,9,11,10,12,15,16,13,18,26]+[5,17,18]+[1,19,20]
48	19	3	[2,3,4,6,7,8,9,10,11,12,13,14,15,16,18,19,20,22,31]+[5,21,22]+[1,23,24]

Table 24: MCGNDs for $v = 2i\,k_1 + 14$, $k_1 \pmod 4 \equiv 1$, $k_2 = 4$, $i \pmod 4 \equiv 1$, $m \pmod 4 \equiv 3$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
24	5	4	[1,3,4,5,11]+[2,6,7,9]+[10,10,12,16]
64	5	4	[4,9,10,11,30]+[14,25,28,29,32]+[7,12,13,15,17]+[5,6,16,18,19]+[20,21,22,24,41]+[1,2,30,31]+[3,8,26,27]
32	9	4	[2,3,4,5,6,7,8,13,16]+[1,9,10,12]+[14,14,15,21]
40	13	4	[2,3,4,6,7,8,9,11,10,13,15,20,12]+[17,18,19,26]+[1,5,16,18]
48	17	4	[4,5,6,7,9,10,12,11,13,14,15,16,20,22,24,21,31]+[3,8,18,19]+[1,2,22,23]

Table 25: MCGNDs for $v = 2i\,k_1 + 14$, $k_1 \pmod 4 \equiv 3$, $k_2 = 4$, $i \pmod 4 \equiv 3$, $m \pmod 4 \equiv 3$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
56	7	4	[1,3,4,5,6,9,28]+[10,11,12,13,14,25,27]+[18,19,22,23,24,26,36]+[8,15,16,17]+[2,7,21,26]
80	11	4	[1,2,3,5,6,7,8,9,10,11,18]+[12,15,16,17,19,20,21,22,27,40,31]+[14,23,24,26,28,30,32,33,34,37,39]+[35,36,38,51]+[4,13,25,38]

Table 26: MCGNDs for $v = 2i\,k_1 + 18$, $k_1 \pmod 4 \equiv 1$, $k_2 = 5$, $i \pmod 4 \equiv 3$, $m \pmod 4 \equiv 3$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
48	5	5	[1,2,4,20,21]+[7,8,9,10,14]+[3,6,11,13,15]+[12,16,18,19,31]+[5,22,22,23,24]
88	5	5	[1,2,14,30,41]+[7,8,9,24,40]+[11,12,13,17,35]+[15,16,18,19,20]+[4,10,21,25,28]+[3,6,23,27,29]+[22,31,33,34,56]+ [26,36,37,38,39]+[5,42,42,43,44]
72	9	5	[1,2,4,5,6,7,8,9,30]+[10,11,12,13,14,15,16,17,36]+[3,19,21,22,23,24,27,31,46]+[20,25,32,33,34]+[18,28,29,34,35]
96	13	5	[1,2,3,4,5,6,7,8,9,10,11,12,18]+[13,15,16,17,19,20,21,22,23,24,25,26,47]+ [28,29,30,31,32,36,33,37,38,39,44,42,61]+[27,34,40,43,48]+[14,41,45,46,46]

Table 27: MCGNDs for $v = 2i\,k_1+18$, $k_1 \pmod 4) \equiv 3$, $k_2 = 5$, $i \pmod 4) \equiv 1$, $m \pmod 4) \equiv 3$ and $v \leq 100$

v	k_1	k_2	Sets of Shifts
32	7	5	[1,2,3,5,6,7,8]+[4,12,13,14,21]+[9,10,14,15,16]
88	7	5	[1,2,3,4,7,30,41]+[8,10,11,12,13,14,20]+[15,16,18,19,21,31,56]+[22,23,24,25,26,27,29]+[6,9,17,33,34,37,40]+ [28,35,36,38,39]+[5,42,42,43,44]
40	11	5	[1,2,3,4,6,7,10,9,17,8,13]+[11,12,15,16,26]+[5,18,18,19,20]
48	15	5	[1,2,3,4,6,7,8,9,10,11,13,15,14,20,21]+[12,16,18,19,31]+[5,22,22,23,24]
56	19	5	[1,2,3,4,5,7,8,9,11,13,12,14,15,16,17,18,24,26,19]+[10,21,22,23,36]+[6,25,26,27,28]

5. Conclusion

As MCGNDs are considered to be the good alternate to the minimal circular neighbor designs for *v* even. These designs are economical to minimize the bias due to the neighbor effects. In this article, therefore, some generators are developed which produce sets of shifts for MCGNDs in blocks of two different sizes. Through these generators, catalogues of the MCGNDs are compiled for *v* (even) ≤ 100. These catalogues are very important for the experimenters as these provide the readymade solution.

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