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On the elliptical orbit of the Earth and position of the Sun in the sky: an engineering approach

Ruben Avila and S. Shoaib Raza*

Department of Thermo-fluids, Faculty of Engineering, National Autonomous University of Mexico (UNAM), Mexico D.F., C.P. 04510, Mexico

ABSTRACT

The position of the Sun as seen by an observer on the Earth's surface and the position and velocity vectors of the Earth revolving in an elliptical orbit around the Sun can be calculated using several computational approaches. These approaches include (but are not limited to) the use of an analytical approach; a numerical approach, and the use of a Solar Position Algorithm (PSA). In the analytical methodology, the Earth's momentum equation is transformed to eliminate its time dependence, and the equation is solved analytically. Whereas, using the numerical approach, the dimensionless momentum equation of the revolving Earth is written in the polar coordinate system (r, θ) and solved numerically. The solar position algorithm known as PSA (Plataforma Solar de Almeria, abbreviated from its Spanish origin: https://www.psa.es), is a numerical algorithm that uses several empirical relations to calculate the solar declination and the ecliptic longitude angles, etc. The algorithm uses Cartesian coordinate system to calculate the dimensionless coordinates of the pole star (Polaris) and its declination angle to calculate the position vector of an observer that rotates with the Earth. This coordinate system is referred to as a new Cartesian coordinate system whose origin is located at the center of the Earth. The solar elevation angle and azimuth angle are obtained by performing a set of rotations of this new Cartesian coordinate system. In this article, we have used basic physical principles (analytical approach) to obtain the main parameters of the Sun's trajectory and position, at certain time in the sky. The methodology presented here can easily be used by professionals and engineers working in the area of solar/alternate energy, as well as for the design of intelligent/green buildings/cities for a sustainable environment.

Keywords: *Position of Sun; Solar Trajectory; PSA; Declination Angle; Orbit of Earth*

1. Introduction

The variation in the position of the Sun in the sky over an observer is a natural phenomenon that has intrigued humankind forever. The position of the Sun has been correlated with the occurrence of natural phenomena (volcanic activity, storm cycles, and earthquakes). The motion of the Sun has also been considered as a measure of time, or as a phenomenon that governs the agricultural cycles and diseases. Since the 20th century, the accurate determination of the Sun's position has been an important subject of study for engineers. Due to the increasing price of petroleum, the effect of greenhouse gases on climate change and global warming, and the increasing number of internal combustion engines in big cities, it is necessary that engineers develop alternative energy sources. Engineers need to efficiently extract energy from renewable and free sources, such as the Sun. Energy engineers need to design efficient solar furnaces, solar steam generators, solar water heaters, solar cells, etc. If civil engineers efficiently use solar energy, they may design reliable intelligent buildings and sustainable environments. In the near future, the task of the engineers will be very important. However, sometimes they do not have enough background to understand the mathematical notations that the physicists and astronomers use to calculate the Sun's position. In view of the above, we have made a literature review and explained the methodology, here.

The determination of the Sun's position in the sky by using vector analysis techniques has been previously reported by [1] and [2]. A simple parametric model, that describes the basic principles of the visible Sun's path on the celestial sphere, has been presented by [3]. A review of the Sun's position algorithms that were published in the solar literature is presented by [4]. The Sun position algorithms are sophisticated schemes, that compute the position of the Sun in the ecliptic, celestial, and horizontal coordinates, see [5]. Very recently, a review of the Sun position algorithms has been presented in [6]. On the internet sites, it is also possible to find and execute computer codes to calculate the position of the Sun in the sky, for instance [7]. The purpose of this paper is to present a self-contained material suitable for energy and civil engineers/researchers to determine the solar position in the sky.

The paper is organized as follows. In section 2, the Earth's orbit equation is presented. In section 3, the methodology to obtain the Cartesian coordinates of the star Polaris and the calculation of the declination angle, are presented. In section 4, a Cartesian coordinate system, whose origin is located at the center of the Earth, is introduced to define both the position vector of an observer and the position vector of the Sun. Employing a set of rotations, the solar elevation angle and the solar azimuth angle measured from the north are calculated. Discussion on results and Conclusions are presented in sections 5 and 6, respectively.

2. Earth's orbit equation

In the mathematical model of the Earth's orbit equation, it is assumed that the Earth is being attracted to a fixed attracting focus (the Sun). The motion is confined to the ecliptic plane, which is described by the radius vector (from the Sun to the Earth) and the velocity vector of the Earth. Using a polar coordinate system r - θ, where r (the radial coordinate) is measured from a fixed focus (the Sun) and θ (the angular coordinate) is measured from a fixed reference line (the line traced from the Sun to the Earth at the Perihelion position), see Fig. 1.

*Corresponding author: ssraza@msn.com

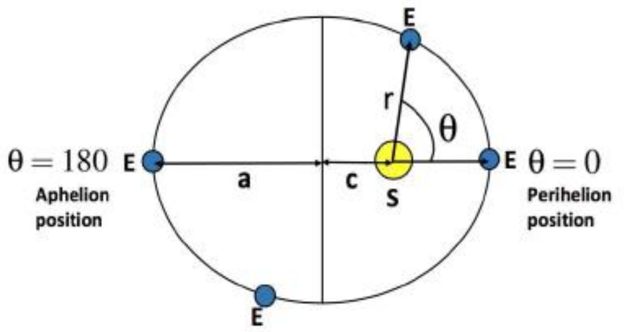

Fig. 1. Polar r - θ coordinate system. The letters S and E correspond to the Sun and the Earth positions, respectively. a and c represent the major axis and the focus of the elliptical trajectory of the Earth around the Sun, respectively. The Earth's Perihelion position (θ=0°) and the Earth's Aphelion position (θ=180°) are also shown.

The radial and angular components of the momentum equation of the Earth (in terms of force and acceleration) are written as (in the model it is assumed that the only force acting on the Earth is in the negative radial direction).

Radial component

If we assume that the motion of the Earth relative to the Sun, is the same as if the Sun is fixed and the mass of the Earth is replaced by the reduced mass M'_E, which is defined as

$$M'_E = \frac{M_E M_S}{M_E + M_S} \tag{1}$$

where M_S is the mass of the Sun and M_E is the mass of the Earth and the gravitational force in the radial direction (attracting force) is balanced by the centripetal force, we get (G is the gravitational constant):

$$\ddot{r} - r\dot{\theta}^2 = -\frac{G\,(M_E + M_S)}{r^2} \tag{2}$$

or

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu}{r^2} \tag{3}$$

Where μ is the gravitational coefficient (positive constant) given as: $\mu = G\,(M_E + M_S)$. Please note dot (.) represents a time derivative and r and θ are shown in Fig.1.

Angular component

Considering that angular momentum is zero, the angular momentum component per unit mass h, is independent of time and is defined as:

$$h = r^2\dot{\theta} \tag{4a}$$

or (squaring both sides and rearranging)

$$\dot{\theta}^2 = \frac{h^2}{r^4} \tag{4b}$$

The principle of conservation of angular momentum states that the moment of the total external force applied to the Earth is equal to the time rate of change of the angular momentum of the Earth about the Sun. If the external moment is equal to zero, the angular momentum h must be a constant. Substituting Eq. (4b) into Eq. (3), we obtain

$$\ddot{r} - r\frac{h^2}{r^4} = -\frac{\mu}{r^2} \tag{5}$$

or

$$\frac{d^2 r}{dt^2} = \frac{1}{r^2}\left(\frac{h^2}{r} - \mu\right) \tag{6}$$

2.1 *Solution of the Earth's orbit equation: A numerical approach*

Using the non-dimensional variables r* = r/a (where a is the semi-major axis of the Earth's elliptical orbit) and t* = tμ/(ah), the dimensionless radial component of the momentum equation is written as

$$\frac{d^2 r^*}{dt^{*2}} = \left(\frac{h^4}{a^2\mu^2 r^{*3}} - \frac{h^2}{a\mu r^{*2}}\right) \tag{7}$$

For the numerical solution of Eq. (7), we have considered the following values (which are available in the literature) of the Earth trajectory: (i) the semi-major axis a = 150 × 10^9 m, (ii) the angular momentum per unit mass h = 4,452,990,073 km^2/s [8] and (iii) the gravitational coefficient μ is obtained by employing the values of G = 6.673x10^{-11} $m^3/(kg\ s^2)$, M_E = 5.972 x 10^{24} kg and M_S = 2 x 10^{30} kg, as $\mu = G\,(M_E + M_S)$ = 132,774,392,455,423,200,000.0 m^3/s^2.

By substituting these values in Eq. (7), we have

$$\frac{d^2 r^*}{dt^{*2}} = \frac{0.999442337}{r^{*3}} - \frac{0.99972113}{r^{*2}} \tag{8}$$

This second-order ordinary differential equation is solved by using a Runge-Kutta-Nystrom technique (D02LAF-NAG) and taking into account the following initial conditions: (i) at t*=0 the Earth's radial velocity is zero, that is, dr*/dt* = $\dot{r}^*$ = 0 and (ii) at t*=0, the Earth is at the Perihelion position i.e. θ = 0°. The numerical solution is performed along the whole year (365 days), by considering 525,600 time steps, which corresponds to a dimensionless time increment Δt*=1.19023 x 10^{-5}, which is equivalent to a dimensional time increment Δt=60s. By solving Eq. (8), we obtain, as a function of time t*, the dimensionless radial position (r*) and the dimensionless radial component of the velocity vector (dr*/dt* = $\dot{r}^*$). The dimensional tangential velocity of the Earth ($v_t = r\dot{\theta}$), which is called the orbital speed, is obtained from Eq. (4a), which says $v_t = r\dot{\theta} = h/r$.

The angle θ, around the Sun is calculated from the equation of an ellipse, which in polar coordinates seems as:

$$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\theta} = \frac{l}{1+\varepsilon\cos\theta} \tag{9}$$

Where, ε is the eccentricity of the Earth's orbit, currently ε ≈ 0.0167005 and l is the semi-latus rectum defined as l = a (1−ε^2) = 149,958,163,991.09 m. The dimensionless version of Eq. (9) is

$$r^* = \frac{(1-\varepsilon^2)}{1+\varepsilon\cos\theta} = \frac{l^*}{1+\varepsilon\cos\theta} \tag{10}$$

where l^* = (1−ε^2) = 0.9997210932. Hence the angle θ is obtained by rearranging the above equation, as:

$$\theta = \cos^{-1}\left(\frac{l^* - r^*}{r^*\varepsilon}\right) \tag{11}$$

From Eq. (11) it is clear that when θ=0° (defined as the Perihelion position, around January 3rd), the Earth is closest to the Sun, at a dimensional radial distance r equal to r_p, as:

$$r_p = a(1-\varepsilon) \approx 147.5 \times 10^6 \text{ km} \quad (12)$$

while when θ =180° (defined as the Aphelion position, around July 4th), the Earth is farthest from the Sun, at a dimensional radial distance, r equal to r_A, as:

$$r_A = a(1+\varepsilon) \approx 152.5 \times 10^6 \text{ km} \quad (13)$$

Then, the dimensionless polar coordinates at the Perihelion r^*_P and Aphelion r^*_A positions (from Eq. 10) are r^*_P = (1−ε) = 0.9833 and r^*_A= (1+ε) = 1.0167, respectively. From the numerical solution of Eq. (8), we obtain r^*_{Pn}=0.9833 (when θ=0°), r^*_{An}=1.016699 (when θ=180°) and l^*_n=0.999721, where, the subscript n refers to the use of the numerical approach.

2.2 *Solution of the Earth's orbit equation: An analytical approach*

If the above equations are reformulated to eliminate time dependence, the time derivatives of the radial distance, *r* are eliminated, and after applying considerable mathematics (for further details of the procedure please contact the corresponding author) and the use of available data for μ, *h* and *a*, the dimensionless Earth's elliptical orbit equation is obtained as:

$$r^* = \frac{r}{a}$$

$$= 0.99972109732795145 / (1+0.016700379398341621 \cos\theta) \quad (14)$$

If Kepler's second law of planetary motion that states "the radius vector from planet to Sun, sweeps equal areas in equal times as the planet orbits the Sun", we obtain an expression that relates the Earth's angle θ around the Sun to the elapsed time since θ = 0 radians (that is, angle from the Earth's perihelion position). If the small element of the area in the elliptical Earth's orbit consists of a small isosceles triangle whose sides have length *r* and whose base length is $rd\theta$. The small area is given as:

$$dA = \frac{1}{2} r^2 d\theta \quad (15)$$

In a short time *dt* the Earth has the constant areal velocity, given by:

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} \quad (16)$$

After applying a considerable mathematics, using Eq. (3) and the principle of conservation of angular momentum, we obtain:

$$dA = \frac{1}{2}\left[\frac{(1+\varepsilon)^2 r_p^2}{(1+\varepsilon\cos\theta)^2}\right] d\theta \quad (17)$$

and,

$$\frac{1}{2}\left[\frac{(1+\varepsilon)^2 r_p^2}{(1+\varepsilon\cos\theta)^2}\right] d\theta = \frac{1}{2} h dt \quad (18)$$

which can be rewritten (after performing the integration over time) as:

$$t = \frac{(1+\varepsilon)^2 r_p^2}{h} \int_0^{2\pi} \frac{d\theta}{(1+\varepsilon\cos\theta)^2} \quad (19)$$

The dimensional analytical solution of Eq. (19) is

$$t = \frac{(1+\varepsilon)^2 r_p^2}{h} [\frac{2}{\sqrt{1-\varepsilon^2}} \tan^{-1}(\frac{\sqrt{1-\varepsilon^2}}{1+\varepsilon} \tan(\frac{\theta}{2})) - \frac{\varepsilon \sin\theta}{1+\varepsilon\cos\theta}] \quad (20)$$

where the dimensional time, t, is in seconds. In the derivation of the above equations, the values for h=4,456,990,073,000,000 m²/s and a=149,597,885,651 m, have been used. The dimensionless expression for the time *t** is given as:

$$t^* = (\frac{\mu}{ah}) \frac{(1+\varepsilon)^2 r^2{}_p}{h} [\frac{2}{\sqrt{1-\varepsilon^2}} \tan^{-1}(\frac{\sqrt{1-\varepsilon^2}}{1+\varepsilon} \tan(\frac{\theta}{2})) - \frac{\varepsilon \sin\theta}{1+\varepsilon\cos\theta}] \quad (21)$$

From Eqs. (14) and (21), the values of r* and t* respectively, are calculated for a set of θ angles in the interval $0 \le \theta \le 2\pi$.

2.3 *Solar position algorithm (PSA)*

The Sun's position algorithm, namely, "Plataforma Solar de Almeria" (PSA algorithm - abbreviated from its Spanish origin: https://www.psa.es) developed by [4] is a numerical algorithm used to calculate, as a function of time (specified by the Julian day, the calendar date and the universal time), the Sun position parameters, such as the ecliptic longitude angle $\hat{\theta}$ (which is related with the polar coordinate, θ, see the previous sections) and the declination angle, β, among others. Following [4] the difference n, between the current Julian day (*jd*) and Julian day 2,451,545 (which corresponds to the day starting at 12:00 UT on January 1, 2000) is given by:

$$n = jd - 2451545 \quad (22)$$

Where, the current Julian day is obtained from:

$$jd = \frac{1461 * (year + 4800 + jm1412)}{4} + \frac{367 * (month - 2 - 12 * jm1412)}{12} - \left(\frac{3 * (year + 4900 + jm1412)}{100 * 4}\right) + day - 32075 - 0.5 + \frac{h\hat{o}ur}{24} \quad (23)$$

Where, the parameter hôur includes the hour of the day (hour) in Universal Time and in decimal format (that is, the minutes and seconds as a fraction of an hour are also included), then

$$h\hat{o}ur = hour + \left[minutes + \frac{seconds}{60}\right] / 60 \quad (24)$$

And, $jm1412$ = (*month*-14)/12. All divisions except the last one are integer divisions, see [4]. The ecliptic coordinates (the coordinates evaluated on the ecliptic plane) are computed for the required Julian day.

3. Location of the star Polaris (North Star) and the declination angle

To calculate the position of the Sun in the sky of an observer, we direct the Earth's rotation axis to the star Polaris. The coordinates of the North star are defined in a Cartesian coordinate system whose origin is located at the center of the Earth's elliptical orbit and whose plane x_1-x_2 defines the ecliptic plane, see Fig. 2. Where the subscript E, refers to the Earth; N to the North Star and S refers to the position of Sun.

We have assumed (considering that the North Star is far away from the Sun-Earth system) that the angle between the rotation axis of the Earth to the position vector of the North Star is the same as that of its position vector with the ecliptic plane (also see Fig. 3, below).

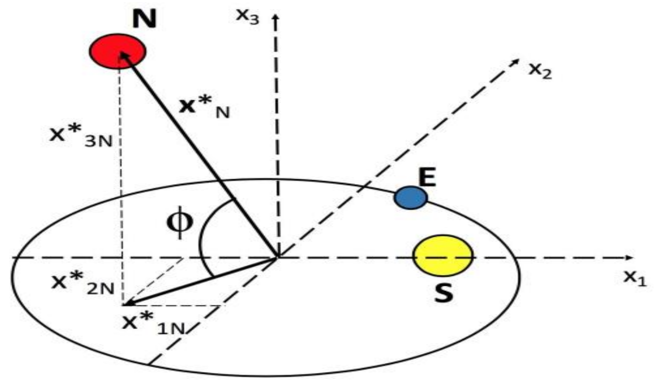

Fig. 2. Cartesian coordinate system, whose origin is located at the center of the Earth's elliptical orbit. In the figure positions of the Sun, Earth and the North Star (Polaris) are shown. Also shown is the angle (66.5477°) between the position vector of Polaris and the ecliptic plane (plane x_1–x_2 of the Cartesian coordinate system).

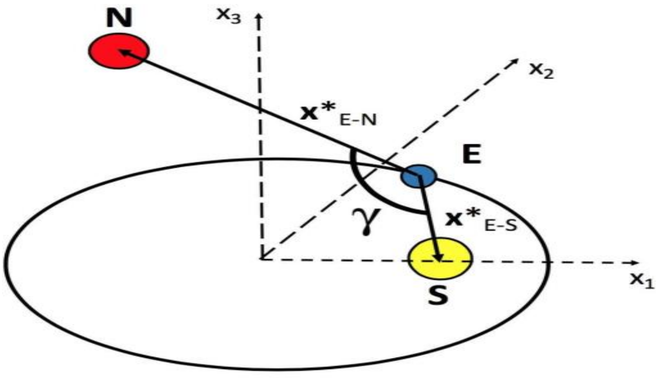

Fig. 3. The angle γ, between the Earth's rotation vector and the vector from the Earth to the Sun.

The angle γ, between the Earth's rotation vector x_{E-N}^* and the vector from the Earth to the Sun $x_{E-S}^* = x_S^* - x_E^*$, is obtained as (see also Fig. 4, below). Its components are:

$$x_{1E-S}^* = -r^* cos\theta, \;\; x_{2E-S}^* = -r^* sin\theta, \;\; x_{3E-S}^* = 0 \quad (25)$$

$$\gamma(t) = \cos^{-1}\left(\frac{x_{E-S}^* \, . \, x_{E-N}^*}{|x_{E-S}^*||x_{E-N}^*|}\right) \quad (26)$$

The declination angle β (t) between the Earth's equator and the vector from the Earth to the Sun x_{E-S}^* is given as (see Fig. 4, below).

$$\beta(t) = 90^o - \gamma(t) \quad (27)$$

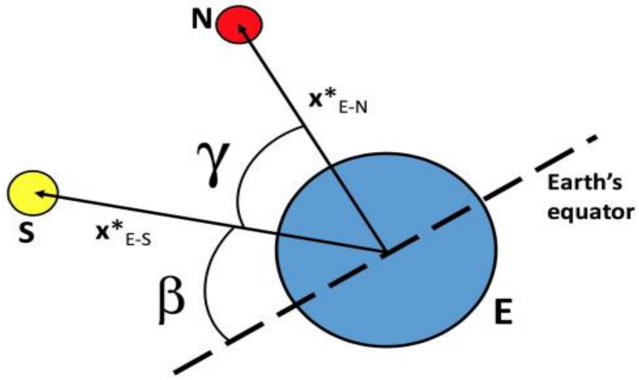

Fig. 4. The declination angle β between the Earth's equator and the vector from the Earth to the Sun x_{E-S}^*.

By performing a trial and error procedure, the North Star is calculated to be at the dimensionless coordinates $x_{1N}^* = 4 * 10^9$ and $x_{2N}^* = -1 * 10^9$

The third component of the position vector of the North Star is calculated as:

$$x_{3N}^* = (\, x_{1N}^{*2} + \, x_{2N}^{*2})^{\frac{1}{2}} \tan(\phi) = 9.5 * 10^9 \quad (28)$$

The convergence criteria of the trial and error process are based on the successful evaluation of the dates at which the equinoxes and solstices occur.

Fig. 5 shows the declination angle β and the ecliptic longitude $\hat{\theta}$ as functions of the days along the year. In the numerical solution (left panel), β is calculated from Fig. 4, while θ is calculated from Eq. (11) (and converted to the ecliptic longitude $\hat{\theta}$). In the analytical solution (middle panel), θ is the independent variable of Eq. (20) (and it is converted to the ecliptic longitude $\hat{\theta}$). In the Sun position algorithm (right panel), PSA is used to calculate $\hat{\theta}$. The calculations have been made for the year 2013. Note that for the three approaches, at the equinoxes, the declination angle β is equal to zero hence at the Earth's equator a vertical zenith is reached. While at the summer and winter solstices, the vector x_{E-S}^* passes through the Tropic of Cancer (i.e. β = 23.45°) and Tropic of Capricorn (i.e. β = - 23.45°) respectively.

The dimensionless tangential velocity (orbital speed) of the Earth that is calculated by the numerical and analytical algorithms is depicted in Fig. 6.

4. The position vector of an observer on Earth and the Earth's rotation

In order to consider the two motions of the Earth: (i) rotation about its axis that points towards the North star and (ii) the elliptical trajectory around the Sun, a new fixed Cartesian coordinate system (o, $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$) is defined, see Fig.7.

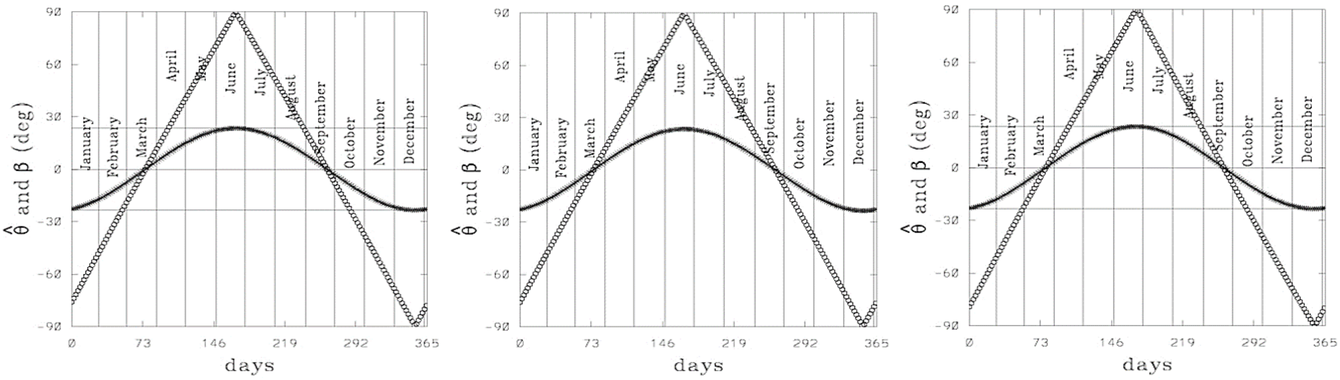

Fig. 5. The declination angle β (*) and the ecliptic longitude $\hat{\theta}$ (◦) as functions of time (days along the year). Left panel: Numerical solution, β is calculated from Fig. (4), while θ is calculated from Eq. (11) (and converted to the ecliptic longitude $\hat{\theta}$). Middle panel: Analytical solution: θ is the independent variable of Eq. (20) (and it is converted to the ecliptic longitude $\hat{\theta}$). Right panel: Sun position algorithm (PSA) is used to calculate $\hat{\theta}$.

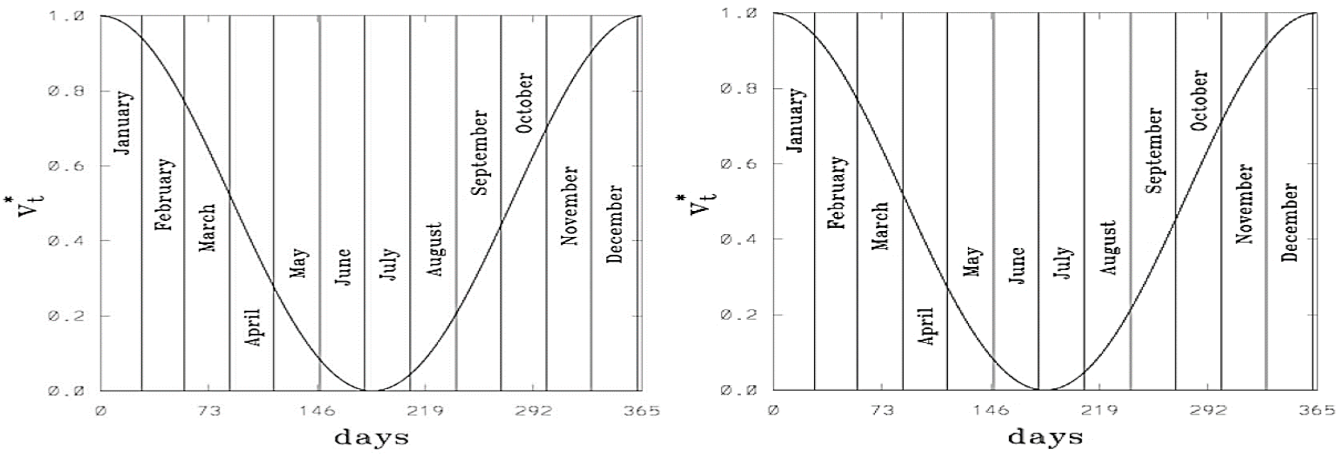

Fig. 6. Dimensionless tangential velocity (orbital speed) of the Earth, $v^*_t = (v_t - v_{tmin})/(v_{tmax} - v_{tmin})$ as a function of time (days along the year). The dimensional tangential velocity is calculated from Eq. (4), as $v_t = r\dot{\theta}$). Left panel: Numerical solution, r (which is the dimensional value of r*) is calculated from Eq. (8) (v_{tmax} = 30252.76 m/s and v_{tmin}= 29258.89 m/s). right panel: Analytical solution, r (which is the dimensional value of the variable $r^* = r/a$, where a is the semi-major axis of the Earth's elliptical orbit) is calculated from Eq. (14) (v_{tmax}= 30299.14 m/s and v_{tmin}= 29303.75 m/s).

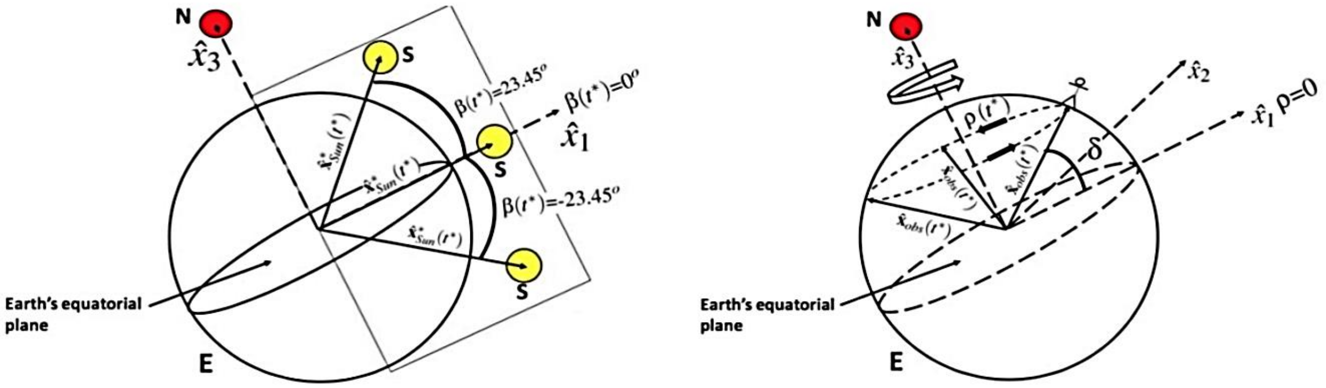

Fig.7. Cartesian coordinate system o, $\hat{x}_1$, $\hat{x}_2$, $\hat{x}_3$ whose origin is located at the center of the Earth. Its $\hat{x}_3$ axis points towards the star Polaris and its plane $\hat{x}_1$ - $\hat{x}_2$ is on the Earth's equatorial plane. Left panel shows that the position vector of the Sun $\hat{x}*Sun(t^*)$ moves on the plane $\hat{x}_1$ - $\hat{x}_3$. Right panel shows the position vector of an observer $\hat{x}_{obs}(t^*)$, that is located at a certain fixed latitude δ on the Earth's surface and the rotation angle ρ.

It may be noted that $t^* = 0$ at $\rho = 0$

This coordinate system has the following characteristics: (i) its origin is located at the center of the Earth, (ii) its plane $\hat{x}_1$ - $\hat{x}_2$ is on the Earth's equatorial plane, (iii) the position vector of the Sun moves on the plane $\hat{x}_1$ - $\hat{x}_3$, (iv) its $\hat{x}_3$ axis points towards the star Polaris and (v) the orientation of its $\hat{x}_1$ axis is defined together with the initial value (at $t^* = 0$) of the rotation angle ρ, we have assumed that at $t^* = 0$, $\rho = 0$ radian. In this new Cartesian coordinate system, we define two vectors, the vector $\hat{x}_{obs}(t^*)$, which is the position vector of an observer that is located at a certain fixed latitude δ on the Earth's surface, and the vector $\hat{x}^*_{sun}(t^*)$, which is the Sun's position vector. Notice that the vector $\hat{x}_{obs}(t^*)$ rotates at the same angular velocity as the Earth, see right panel of Fig. 7. In the model, it is assumed that the Earth's rotation angle ρ is $0 \leq \rho \leq 2\pi$, where 2π radian, corresponds to 1 day (24 hours or 86400 seconds). The increment of the rotation angle Δρ (which corresponds to the time step Δt=60 s of the numerical solution) is calculated as:

$$\Delta\rho = \frac{2\pi * 60}{24 * 3600} = 0.00436\, radians \tag{29}$$

The dimensionless three components of the rotating vector $\hat{x}^*_{obs}(t^*)$ referred to as the fixed Cartesian coordinate system are given as:

$$\hat{x}^*_{1obs}(t^*) = \cos\delta \cos\rho(t^*),$$

$$\hat{x}^*_{2obs}(t^*) = \cos\delta \sin\rho(t^*)\,, \hat{x}^*_{3obs}(t^*) = \sin\delta \tag{30}$$

while the dimensionless three components of the Sun's position vector $\hat{x}^*_{1sun}(t^*)$, which oscillates from $\beta(t^*) = -23.45^o$ to $\beta(t^*) = 23.45^o$ on the plane $\hat{x}_1$ - $\hat{x}_3$, are the following:

$$\hat{x}^*_{1sun}(t^*) = cos\beta(t^*), \hat{x}^*_{2sun}(t^*) = 0, \hat{x}^*_{3sun}(t^*) = sin\beta(t^*)$$

It may be noted that in Eq. (30), the dimensionless radius of the Earth is taken as equal to 1.

5. Results and Discussion

Some of the preliminary results obtained using these computational methodologies viz., the Numerical approach, Analytical approach, and the PSA are presented in Figures 5 and 6. Fig. 5 shows the declination angle β and the ecliptic longitude $\hat{\theta}$ as functions of the 365 days along the year 2013 (chosen arbitrarily to demonstrate the methodology). Both the numerical and analytical solutions compare well with the PSA which demonstrates the suitability and validity of our computational methodology. It may be noted in Fig. 5, that at the time of equinoxes, the declination angle β is equal to zero hence at the Earth's equator, a vertical zenith is reached. While at the summer and winter solstices, the vector x^*_{E-S} passes through the Tropic of Cancer (i.e. $\beta = 23.45^o$) and Tropic of Capricorn (i.e. $\beta = -23.45^o$), respectively.

The tangential velocity (orbital speed) of the Earth is calculated by the numerical and analytical algorithms. It can be observed in the figure (see Fig. 6) that the dimensionless tangential velocity of Earth predicted by both the methodologies compare well, hence elaborating the success of our computations.

6. Conclusions

Diverse computational methodologies have been presented to calculate the trajectory of the Sun in the sky of an observer located on the Earth's surface. A numerical algorithm and an analytical methodology have been used to get the parameters needed to obtain the Sun's position in our sky. The location of the North Star has been calculated in a Cartesian coordinate system, which is a familiar coordinate system for engineers. Additionally, for the calculation of the location of the North Star, the position vector from the Earth to the Sun γ and the declination angle β as a function of time, were obtained for the use of the energy engineers. Standard transformations of the involved vectors (the position vector of the observer and the Sun's position vector) have been obtained by performing simple rotations of the Cartesian coordinate system.

The information included in this paper, although is a standard one, should be considered as an important source of reference, for solar energy engineers/civil engineers. For the construction of intelligent buildings for a sustainable environment, engineers may use this approach to accurately know the position of the Sun in the sky throughout the year.

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