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LRS Bianchi Type-I Cosmological Model in Modified *f(R, T)* Gravitation Theory filled with Perfect Fluid

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ABSTRACT

This study explores LRS Bianchi Type-I space time filled with a perfect fluid within the framework of f(R,T) gravity, where R represents the Ricci scalar and T denotes the trace of the stress-energy momentum tensor. We analyze the simplest form of cosmic evolution in the context of general non-minimally coupled gravity models. Two specific models of f(R,T) gravity are considered. A time-dependent deceleration parameter is introduced, leading to an accelerated universe with an exact field solution. Additionally, we examine the kinematical and physical properties of the proposed models.

Keywords: *LRS Bianchi type-I; f(R,T) gravity; Perfect fluid.*

1. Introduction

It was made possible by the revolution in current cosmological understanding because of the observational cosmology research during the last two decades. Currently, the results of observational studies imply that the universe is growing more quickly than before [1-10]. Recent information provided by the Plank collaboration [11], Baryon Oscillation Spectroscopic Survey (BOSS) [12] and Atacama Cosmology Telescope Polarimeter (ACTPol) Collaboration [13] gives essential experimental data supporting that universe is in accelerated expansion stage. Moreover, the high red shift supernova experiments (HRSSE)[14,15] give clear indication that the indirect evidence for cosmic acceleration originates with observations like the cosmic microwave background (CMB) fluctuation [4] and large scale structure (LSS) [5].

There are two primary approaches for solving the challenge of cosmic acceleration. Introducing a cosmic component of dark energy is the first step and looks into its dynamic behavior and modifying general relativity is the second technique itself. Both strategies have unique elements as well as some serious theoretical issues. However, in this paper, our goal is to modify gravity, general relativity has seen various modifications in the last few decades.

Astrophysical measurements demonstrate that the universe is expanding rapidly due to an unusual form of energy accompanied by a strong negative pressure, known as dark energy, despite observational evidence. A tricky problem is still the character of dark energy in modern cosmology. The mysterious nature of dark energy is explained by modified theories of gravitation. Consequently, late time acceleration has been investigated by researchers and Dark energy can be investigated with modifying general relativity (GR) i.e. through modifying geometric part of Einstein–Hilbert action [16]. A highly successful strategy across them is this one to explore dark energy. The negative pressure created by "dark energy" and therefore causes the Universe to expand faster than the usual. Considering the Wilkinson Microwave Anisotropy Probe (WMAP) satellite experiment, the dark energy occupies 73% of the matter in the universe is non-baryonic dark matter that fills up to 23% and regular baryonic (normal) matter occupy 4%. Cosmologists have postulated different types of dark energy candidates, including the cosmological constant, the Tachyon, the quintessence, the phantom, and others, to explain the observed data.

It's possible that modifying the Einstein-Hilbert action is going to be the best way to give an explanation how the cosmos has evolved. Among them, *f(R)* suggested a theory of gravity by Nojiri and Odintsov [17], a theory of gravity is remarkable. *f (R, T)* modified theory of gravity was recently created by Harko *et al.* [18], where the stress-energy tensor's trace T and Ricci scalar R's arbitrary function R yield the gravitational Lagrangian. In addition, for test particles the equations of motion, have the metric formalism's equations for the gravitational field, which result from the stress-energy tensor's covariant divergence.

Now, taking into account metric-dependent Lagrangian density, according to the following, the relevant gravity field equations are obtained with the Hilbert-Einstein variations principle. The action in light of $f(R,T)$ theory of gravity is

$$S = \int\left(\frac{1}{16\pi G} f(R,T) + L_m\right)\sqrt{-g}\, d^4x \qquad (1)$$

Here L_m this is the usual matter Lagrangian density of matter source, a random function of Ricci scalar R & the trace T of the energy–momentum tensor T_{ij} the origin of the matter is $f(R,T)$, the determinant of the metric tensor g_{ij} is g. The energy–momentum tensor T_{ij} with the Lagrangian matter is outlined in such a manner and $T = g^{ij}T_{ij}$ is its trace, Here, matter Lagrangian L_m relies only on the metric tensor component g_{ij} in place of its derivatives this is we considered here, Hence, we secure

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,L_m)}{\delta g^{\mu\nu}} \qquad (2)$$

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$$T_{ij} = g_{ij}L_m - \frac{\partial L_m}{\partial g^{ij}} \tag{3}$$

The *f(R, T)* gravitation field's equations are acquired by varying the action *S* in relation to metric tensor ($g_{\mu\nu}$).

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + [g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j]f_R(R,T) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\theta_{ij} \tag{4}$$

here

$$f_R = \frac{\delta f(R,T)}{\delta R},\ f_T = \frac{\delta f(R,T)}{\delta T},\ \Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}.$$

Here ∇ referred as the covariant derivative also T_{ij} are usual matter energy-momentum tensor obtained from the Lagrangian L_m. It is stated here that the physical properties of the matter field are determined by field equations. Numerous theoretical frameworks that represent various contributions of matter for *f(R, T)* gravitational potential; But Still, Harko*et al.*[18] provided three classes for these models $f(R,T) = f_1(R) + f_2(T)$.

$$f(R,T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R)f_3(T) \end{cases} \tag{5}$$

Separate equation of field for different models of $f(R,T)$ gravitation is presented as

1. $f(R,T) = R + 2f(T)$

$$R_{ij} - \frac{1}{2}Rg_{ij} = 8\pi T_{ij} - 2f'(T)T_{ij} - 2f'(T)\theta_{ij} + f(T)g_{ij} \tag{6}$$

2. $f(R,T) = f_1(R) + f_2(T)$

$$f_1'(R)R_{ij} - \frac{1}{2}f_1(R)g_{ij} + [g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j]f_1'(R) = 8\pi T_{ij} - f_2'(T)T_{ij} - f_2'(T)\theta_{ij} + \frac{1}{2}f_2(T)g_{ij} \tag{7}$$

$$\text{If } L_m = p \text{ then } \theta_{ij} = -2T_{ij} - pg_{ij} \tag{8}$$

The selection of the *f (R, T)* model affects the outcome, as is evident. In order to meaningfully depict our results, we must therefore select a workable *f(R, T)* model. The possibly cosmological criteria within *f(R)* theory, which characterizes the dark energy models, have been addressed by Nojiri and Odintsov [19]. The model that Sharif and Zubair [20] have chosen for us to discuss

$$f(R,T) = \alpha_1 R^m T^n + \alpha_2 T(1 + \alpha_3 T^p R^q) \tag{9}$$

Whereas *m, n, p,* and *q* are assumed to have values higher than or equal to 1. We will examine our findings in light of various applications of the above model, and we'll concentrate our subsequent discussion on the next three scenarios.

$$f(R,T) = \alpha_1 R + \alpha_2 T + \alpha_4 T^2 \text{ for } m=1, n=0,\ \alpha_4 = \alpha_1\alpha_3,\ p=1, q=0 \tag{10}$$

$$f(R,T) = R + \alpha_2 T \text{ for } \alpha_1 = 1,\ m=1, n=0,\ \alpha_3 = 0 \tag{11}$$

$$f(R,T) = \alpha_1 R + \alpha_2 T(1 + \alpha_3 T\ \ R^2) \tag{12}$$

Making use of equations (6), (7) and (8) in conjunction with the (10), (11) and (12) , we have

Model-I:- $f(R,T) = \alpha_1 R + \alpha_2 T + \alpha_4 T^2$ *for* $m=1, n=0,\ \alpha_4 = \alpha_1\alpha_3, p=1, q=0$ *is*

$$R_{ij} - \frac{1}{2}Rg_{ij} = \frac{8\pi}{\alpha_1}T_{ij} + \left[\frac{\alpha_2}{\alpha_1} + 2\alpha_3 T\right][T_{ij} + pg_{ij}] + \frac{1}{2}\left[\frac{\alpha_2}{\alpha_1}T + \alpha_3 T^2\right]g_{ij} \tag{13}$$

Model-II:- $f(R,T) = R + \alpha_2 T$ *for* $\alpha_1 = 1,\ m=1, n=0,\ \alpha_3 = 0$

$$R_{ij} - \frac{1}{2}Rg_{ij} = [8\pi + \alpha_2]T_{ij} + \left[p\alpha_2 + \frac{1}{2}\alpha_2 T\right]g_{ij} \tag{14}$$

So many scientists created cosmological models with perfect fluid substance explore the universe's accelerated expansion. Most recently discoveries show that the universe is expanding more quickly than predicted by the negative pressure caused by an unidentified type of energy i.e. named as dark energy. Due to this, we have to build a cosmological model of the accelerating universe lacking consideration for account of dark energy or dark matter, even though also selecting the greatest trustworthy matter component. As a result, the literature has extensively studied numerous cosmological models that include fluid with viscosity in the early universe [21, 22]. Also most of the researcher have studied on *f(T), f(R,T)* gravity [23, 24].

The current document is structured as follows: In Section 2, in this part we extract the exact solutions to one of the instances where *f(R,T)* gravity by using methodology [22]. The perfect fluid model offers a sophisticated explanation of matter behavior, distinguished by its pressure and energy density. It is a perfect fit for our study due of its adaptability and suitability for a variety of physical environments. Regarding the time-varying DP-supported spatially homogeneous anisotropic Bianchi Type-I space-time the bulk viscous pressure, bulk viscous coefficient, energy density, matter trace, Ricci scalar, and energy conditions are obtained. Section 3 & 4 presents the physical properties of both models.

2. Metric, Field Equations Solutions

Cosmological models of the Bianchi type are significant because they are homogenous and anisotropic, providing a framework for studying the universe's isotropization throughout time. Furthermore, anisotropic universes are more general than isotropic models from a theoretical perspective. Bianchi space times are helpful in building models of spatially homogenous &anisotropic cosmologies because of the ease of solving an field equations with their relative simplicity.

The LRS Bianchi Type-I line element is

$$ds^2 = -dt^2 + A^2dx^2 + B^2(dy^2 + dz^2) \tag{15}$$

However, A &B are the scale factors with function of cosmic time *t* only.

The stress energy tensor of matter is taken to be

$$T_{ij} = (p + \rho)u_i u_j + pg_{ij} \tag{16}$$

Whereas $u_i = (0,0,0,1)$ is the four-velocity vector in co-moving coordinate system satisfying $u_i u_j = -1$.

2.1 Model-I

$$f(R,T) = \alpha_1 R + \alpha_2 T + \alpha_4 T^2 \text{ for}$$
$$m = 1, n = 0, \ \alpha_4 = \alpha_1\alpha_3, p = 1, q = 0$$

From (13) and (15) field equation obtained as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = X_1 p + X_2 p^2 + X_3 p\rho + X_4 \rho + X_5 \rho^2 \quad (17)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = X_1 p + X_2 p^2 + X_3 p\rho + X_4 \rho + X_5 \rho^2 \quad (18)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = Y_1 p + Y_2 p^2 + Y_3 p\rho + Y_4 \rho + Y_5 \rho^2 \quad (19)$$

Where dot (.) indicate the derivative with to *t* and

$$X_1 = \frac{8\pi}{\alpha_1} + \frac{7\alpha_2}{2\alpha_1}, X_2 = \frac{33\alpha_3}{2}, X_3 = -7\alpha_3, X_4 = -\frac{\alpha_2}{2\alpha_1}, X_5 = \frac{\alpha_3}{2} \quad (20)$$

$$Y_1 = \frac{5\alpha_2}{2\alpha_1}, Y_2 = \frac{21\alpha_3}{2}, Y_3 = -11\alpha_3 \ Y_4 = -\frac{8\pi}{\alpha_1} - \frac{3\alpha_2}{2\alpha_1} \ Y_5 = \frac{5\alpha_3}{2} \quad (21)$$

$$\frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{2\dot{B}^2}{B^2} = (X_1 + Y_1)p + (X_2 + Y_2)p^2 + (X_3 + Y_3)p\rho + (X_4 + Y_4)\rho + (X_5 + Y_5)\rho \quad (22)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{3\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = (X_1 + Y_1)p + (X_2 + Y_2)p^2 + (X_3 + Y_3)p\rho + (X_4 + Y_4)\rho + (X_5 + Y_5)\rho \quad (23)$$

Equating (22) and (23) we obtain

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{A}}{A} = 0 \quad (24)$$

The expansion scalar relation is proportional to the shear scalar, resulting in

$$A = B^n \quad (25)$$

Using (25) in (24) we have the equation

$$\frac{\ddot{B}}{B} + (1+n)\frac{\dot{B}^2}{B^2} = 0 \quad (26)$$

Assume $\dot{B} = G(B)$ then equation (26) is

$$\frac{dG}{dB} + (1+n)G = 0 \quad (27)$$

This is the linear differential equation in G, which leads the solution

$$G = k_1 e^{-(1+n)B} \quad (28)$$

Where k_1 the constant of integration, hence is the solution is obtained as

$$ds^2 = -\left(\frac{dt}{dB}\right)^2 dB^2 + B^{2n}dx^2 + B^2(dy^2 + dz^2) \quad (29)$$

This is written as

$$ds^2 = -k^2{}_1 e^{2(1+n)B} dB^2 + B^{2n}dx^2 + B^2(dy^2 + dz^2) \quad (30)$$

Now using transformation $B = \tilde{T}$, $x = \tilde{X}$, $y = \tilde{Y}$ and $z = \tilde{Z}$, eq. (30) takes the form

$$ds^2 = -k^2{}_1 e^{2(1+n)\tilde{T}} d\tilde{T}^2 + \tilde{T}^{2n} d\tilde{X}^2 + \tilde{T}^2(d\tilde{Y}^2 + d\tilde{Z}^2) \quad (31)$$

From (31) we have

$$A = \tilde{T}^n \text{ and } B = \tilde{T} \quad (32)$$

The average Hubble parameter obtained as

$$H = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{k_1(n+2)}{3}\frac{e^{-(1+n)\tilde{T}}}{\tilde{T}} \quad (33)$$

Expansion scalar is obtained as

$$\theta = 3H = k_1(n+2)\frac{e^{-(1+n)\tilde{T}}}{\tilde{T}} \quad (34)$$

The value of deceleration parameter will be

$$q = \frac{d}{d\tilde{T}}\left(\frac{1}{H}\right) - 1 = \frac{3e^{(1+n)\tilde{T}}[\tilde{T}(1+n)+1] - k_1(n+2)}{k_1(n+2)} \quad (35)$$

The average scale factor is

$$a = AB^2 = \tilde{T}^{n+2} \quad (36)$$

Shear scalar is obtained as

$$\sigma^2 = \frac{1}{2}\left(H_x^2 + H_y^2 + H_z^2 - \frac{\theta^2}{3}\right) = \frac{k_1^2(n-1)^2}{3} e^{-2(1+n)\tilde{T}} \quad (37)$$

To solve field equation now we consider relation

$$p = \omega\rho \quad (38)$$

From (22) and (38), we get

$$\frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{2\dot{B}^2}{B^2} = T_1\rho^2 + T_2\rho \quad (39)$$

Where

$$T_1 = (X_2 + Y_2)\omega^2 + (X_3 + Y_3)\omega + (X_5 + Y_5),$$
$$T_2 = (X_1 + Y_1)\omega + (X_4 + Y_4) \quad (40)$$

Then from (20) & (21) equation (40)

$$T_1 = 24\alpha_3\omega^2 - 11\alpha_3\omega + 3\alpha_3,$$
$$T_2 = \left[\frac{8\pi}{\alpha_1} + \frac{6\alpha_2}{\alpha_1}\right]\omega - \left[\frac{8\pi}{\alpha_1} + \frac{2\alpha_2}{\alpha_1}\right] \quad (41)$$

Now put value of *A* and *B* from eq. (32) in eq. (39) we get

$$T_1\rho^2 + T_2\rho - \frac{2k_1^2(n+1)(1-\tilde{T})e^{-2(1+n)\tilde{T}}}{\tilde{T}^2} = 0 \quad (42)$$

Equation obtained in (42) is quadratic equation in ρ then by [22]

$$\rho = \frac{-T_2\tilde{T} \pm \left[T_2^2\tilde{T}^2 + 8T_1k_1^2(n+1)(1-\tilde{T})e^{-2(1+n)\tilde{T}}\right]^{1/2}}{2T_1\tilde{T}} \quad (43)$$

$$p = \omega\frac{-T_2\tilde{T} \pm \left[T_2^2\tilde{T}^2 + 8T_1k_1^2(n+1)(1-\tilde{T})e^{-2(1+n)\tilde{T}}\right]^{1/2}}{2T_1\tilde{T}} \quad (44)$$

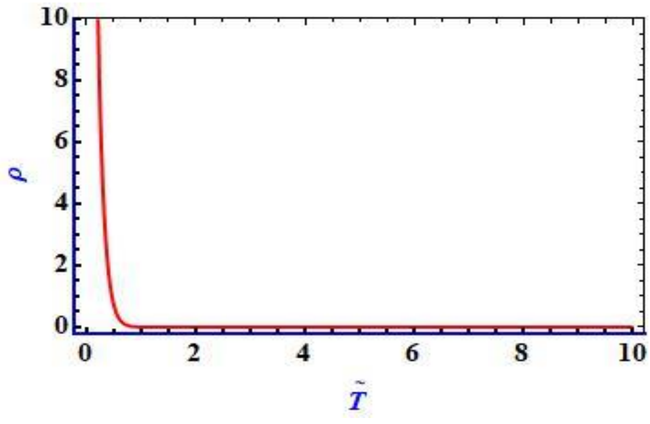

Fig.1. Variation of energy density against $\tilde{T}$ for $k_1 = 5$, $n = 1.5$, $T_1 = 1$, $T_2 = 5$

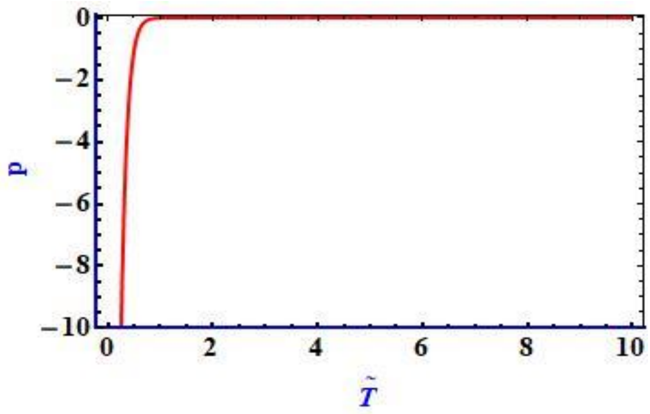

Fig. 2.Variation of pressure against $\tilde{T}$ for $k_1 = 5$, $n = 1.5$, $T_1 = 1$, $T_2 = 5$

2.2 Model-II

$f(R,T) = R + \alpha_2 T$ *for* $\alpha_1 = 1,\ m = 1, n = 0,\ \alpha_3 = 0$.

Using equations (14) and (15), the field equations are obtained as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = [8\pi + \frac{7}{2}\alpha_2]p - \frac{1}{2}\alpha_2\rho \tag{45}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = [8\pi + \frac{7}{2}\alpha_2]p - \frac{1}{2}\alpha_2\rho \tag{46}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = \frac{5}{2}\alpha_2 p - [8\pi + \frac{3}{2}\alpha_2]\rho \tag{47}$$

Here dot represents derivatives with respect to time

In order to resolve the above field equations, we have proceeded in the same manner as describe in Model-I

Now from equation (45) to (47) and (38) we have the equation

$$\frac{2\ddot{B}}{B} + \frac{2\dot{A}\dot{B}}{AB} + \frac{2\dot{B}^2}{B^2} = [(8\pi + 6\alpha_2)\omega - (8\pi + 2\alpha_2)]\rho \tag{48}$$

In this model, by [22] the energy density and pressure are given as

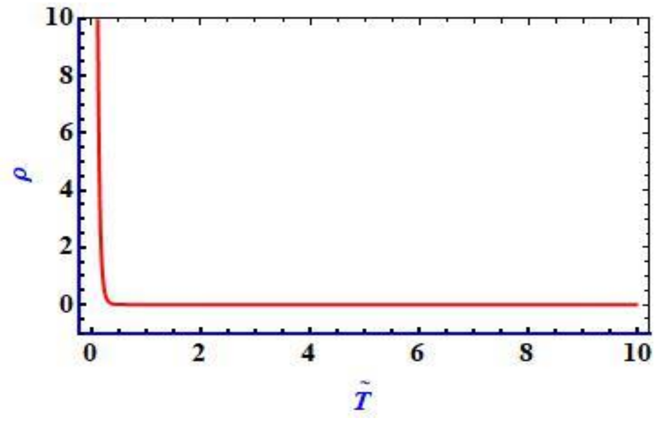

Fig.3. Variation of energy density against $\tilde{T}$ for $k_1 = 2.5$, $n = 5$, $\alpha_2 = 15$, $\omega = 1.5$

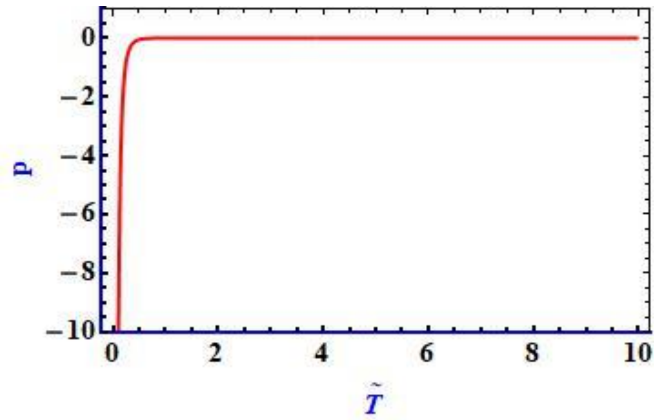

Fig. 4. Variation of pressure against $\tilde{T}$ for $k_1 = 2.5$, $n = 5$, $\alpha_2 = -15$, $\omega = 1.5$

$$\rho = \frac{2k_1{}^2(n+1)(1-\tilde{T})e^{-2(1+n)\tilde{T}}}{\{(8\pi + 6\alpha_2)\omega - (8\pi + 2\alpha_2)\}\tilde{T}^2} \tag{49}$$

$$p = \frac{\omega 2k_1{}^2(n+1)(1-\tilde{T})e^{-2(1+n)\tilde{T}}}{\{(8\pi + 6\alpha_2)\omega - (8\pi + 2\alpha_2)\}\tilde{T}^2} \tag{50}$$

3. State Finder Diagnostic

The state-finder pair {r, s} is characterized as

$$r = 1 + \frac{3\dot{H}}{H^2} + \frac{\ddot{H}}{H^3},\quad s = \frac{r-1}{3(q-\frac{1}{2})} \tag{51}$$

The state-finder pair is diagnostic parameter in geometrical that is directly derived from a space-time metric also has more universal than physical variables because physical variables rely upon the properties of physical fields that define DE.

The state-finder parameter values regarding our examples are

$$r = 1 - \frac{9\tilde{T}^2}{k_1(n+2)}\left\{\frac{1}{\tilde{T}^2} + \frac{(1+n)}{\tilde{T}}\right\}e^{(1+n)\tilde{T}} + \frac{9\tilde{T}^3}{k_1^2(n+2)^2}\left\{\frac{2}{\tilde{T}^3} + \frac{2(1+n)}{\tilde{T}^2} + \frac{(1+n)^2}{\tilde{T}}\right\}e^{2(1+n)\tilde{T}} \quad (52)$$

$$s = \frac{-9\tilde{T}^2\left\{\frac{1}{\tilde{T}^2} + \frac{(1+n)}{\tilde{T}}\right\}e^{(1+n)\tilde{T}}k_1(n+2) + 9\tilde{T}^3\left\{\frac{2}{\tilde{T}^3} + \frac{2(1+n)}{\tilde{T}^2} + \frac{(1+n)^2}{\tilde{T}}\right\}e^{2(1+n)\tilde{T}}}{3e^{k_1(n+2)}\{3e^{(1+n)\tilde{T}}[\tilde{T}(1+n)+1] - \frac{3}{2}k_1(n+2)\}} \quad (53)$$

4. Physical Interpretation of Graph

Fig. 1 shows that for Model-I, As $\tilde{T}$ goes on, the universe's energy density decreases and eventually tends to a constant value 0.1, $\rho \to 0$ as $\tilde{T} \to +\infty$ .and values of density goes to infinite as $\tilde{T}$ tends to zero i.e. $\rho \to +\infty$ as $\tilde{T} \to 0$ so range of ρ is $(0,\infty)$. When it comes to pressure by Fig. 2, it is an increasing function of $\tilde{T}$ and takes the negative values throughout cosmic evolution $p \to -\infty$ as $\tilde{T} \to 0$ and $p \to 0$ as $\tilde{T} \to +\infty$ It starts with extremely high negative values both initially and ultimately gets closer to zero and range of p is $(-\infty,0)$. Recent observations have demonstrated that the Universe is accelerating, and the negative pressure supports this phase and our model.

For Model–II Fig. 3 shows that, a decaying function of time describes the energy density of the universe along with eventually approaches a constant value. i.e. $\tilde{T} \to \infty$ as $\rho \to -0.4$ it will goes very close to y-axis as $\tilde{T} \to 0$ & Range interval of the density is $(-0.5,\infty)$ and from Fig. 4 pressure is increasing function takes the negative values throughout cosmic evolution and range set of the pressure is $(-\infty,0.1)$

Conclusion

In this study, we analyzed the LRS Bianchi Type-I cosmological model within the framework of f(R,T) gravity, considering a perfect fluid as the cosmic source. Two specific f(R,T) models were explored, leading to exact solutions of the modified Einstein field equations under the assumption of a time-dependent deceleration parameter. Various cosmological parameters, including the deceleration parameter, were examined.

By adopting a special law of variation for the Hubble parameter and deceleration parameter, we derived solutions that describe the evolution of the universe. Notably, our results indicate that the deceleration parameter remains negative across all datasets, providing strong evidence that the universe is undergoing accelerated expansion, with the acceleration becoming progressively stronger over time.

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