The Nucleus **54**, No. 3 (2017) 161-163

www.thenucleuspak.org.pk

The Nucleus

ISSN 0029-5698 (Print)

ISSN 2306-6539 (Online)

Reduced Error in Root Mean Square (rms) Value for Square Wave Signal Relative to Sine Wave Signal

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ARTICLE INFO

Article history:

Received : 27 January, 2017

Accepted : 12 September, 2017

Published : 30 September, 2017

Keywords:

Signals,
Mean,
Root mean square,
Error,
Square wave signal,
Parametric control

ABSTRACT

An average responding multimeter measures 11.11% error in root mean square (rms) for square wave signal relative to sine wave signal. In this paper an attempt is made to reduce this error by defining a control on quadratic wave signal. It's limiting value approaches to square wave signal. Then the signal has piecewise continuous structure and the corresponding error in rms value is 3.81%. Hence is more close to its true rms value. It means that this structure of square wave signal is more suitable as compared to its discrete structure.

1. Introduction

Signal is a time varying quantity used to convey information about the behaviour of some phenomenon [1]. The study of signals and systems, particularly linear signals, is a part of many engineering disciplines. In electrical engineering, it may be voltage, current, amount of charge, etc. [2]. The signals are characterized by mean value or root mean square value. Multimeters are designed either they respond to mean value or root mean square value.

Square wave signal is pure DC, which has its own importance in signal processing as it may change very rapidly from its minimum to maximum value. Sinusoidal signal is pure AC, and its rms is true hence its accuracy specifications are applied to all other wave forms. An average responding multimeter measures error in rms for square wave signal as 11.11% relative to sine wave signal [3]. In this paper, an attempt is made to reduce this error by defining a α–parametric control on quadratic wave signal. Here the parameter α, is a real number from the open interval
$0 < \alpha < 1$. For $\alpha \to 0$ the signal approaches to square wave signal and hence has piecewise continuous structure. For this signal the corresponding error in rms is 3.81%. Hence this structure of square wave signal seems to be more suitable as compared to its discrete structure.

2. Mathematics of Signals

In certain applications, it is important to know the strength/size of a signal and this strength/size is measured from mean or *rms* value of a signal [4]. Mathematically a signal may be represented by a function *f(t)*. It may be continuous or discontinuous. Let *f(t)* be a T periodic signal, then its mean value about its period is given by [5].

$$mean = \frac{1}{T}\int_0^T f(t)dt \tag{1}$$

The Root mean square (rms) value is the square root of power, where the power of a signal is:

$$P = \frac{1}{T}\int_0^T f^2(t)dt$$

Root mean square (rms) value is

$$rms = \sqrt{P} = \left(\frac{1}{T}\int_0^T f^2(t)dt\right)^{1/2} \tag{2}$$

For obtaining the AC signal, maximum digital multimeters first perform a full wave reflection of the waveform and then compute its mean value. For rms value a constant factor is multiplied. This constant factor is 1.111. Hence error in rms value of a waveform is

$$error\% = \frac{mean}{rms} \times 11.1107\% \tag{3}$$

3. Parseval's Theorem and Periodic Signals

In 1799 Parseval presented a theorem known as Parseval's theorem, which states that the power of a signal in frequency domain is the same as that of its power in time domain. This theorem uses the concepts of Fourier series/transformation and periodic signals [6]. A periodic signal $f(t)$ of period $T = \frac{2\pi}{\omega}$ has Fourier expansion as [7].

$$f(t) = a_0 + \sum_{k=1}^{\infty}[a_k \cos k\omega t + b_k \sin k\omega t]$$

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Where a_0, a_k and b_k are the Fourier coefficients. Mathematically these coefficients are given by equation (4)

$$\begin{cases} a_0 = \frac{2}{T}\int_0^T f(t)dt; \\ a_k = \frac{2}{T}\int_0^T f(t)\cos k\omega t\, dt; \\ b_k = \frac{2}{T}\int_0^T f(t)\sin k\omega t\, dt; \end{cases} \tag{4}$$

By Parseval's theorem, the power of a signal in frequency domain is [8].

$$P = {a_0}^2 + \frac{1}{2}\sum_{k=1}^{\infty}\left({a_k}^2 + {b_k}^2\right) \tag{5}$$

and the root mean square value of a signal is given by

$$rms = \left({a_0}^2 + \frac{1}{2}\sum_{k=1}^{\infty}\left({a_k}^2 + {b_k}^2\right)\right)^{1/2} \tag{6}$$

Equation (3) involves mean and rms values of a signal. The mean of a signal can be calculated using (1) and rms using eqs. (2) or (6). Here we will use eq. (6) to calculate rms of a signal. Since we are interested in reducing error in rms for square wave signal. The square wave signal is defined in two ways. In first way, it has discrete structure, and in second way it has piecewise continuous structure. First consider square wave signal having discrete structure.

4. Square Wave Signal

A periodic square wave signal $S(t)$ of unit amplitude is defined by equation (7).

$$S(t) = \begin{cases} 1 & if\ 0 \le t < 0.5\,T \\ -1 & if\ 0.5\,T \le t < T \end{cases} \tag{7}$$

and is illustrated in Fig. 1.

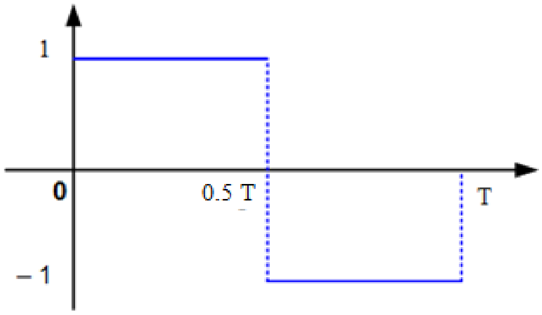

Fig.1: Square wave signal

As the signal is symmetric about T axis, so it's mean is zero. We can take its mean by taking time rate of its area, that is

$$mean = 1$$

To calculate it's rms, we first calculate the Fourier coefficients of (7) that are

$$\begin{cases} a_0 = 0 \\ a_k = 0 \\ b_k = \frac{4}{\pi(2k-1)} \end{cases} \tag{8}$$

Using eq. (8) in eq. (6), the rms associated with this signal is

$$rms = \sqrt{0.5\ \sum_{k=1}^{\infty}\left(\frac{4}{\pi(2k-1)}\right)^2} = 1$$

Also, the power of this signal is 1. The mean, power and rms values of square wave signal are 1 [3, 9].

Using equation (3) the error in rms for square wave signal is

$$error\% = \frac{1}{1} \times 11.1107$$
$$= 11.11\%$$

We are getting rms value 11.11% more than its true value. As square wave signal has jump discontinuity in its structure. There may be a chance to reduce this error if we succeed in removing this jump discontinuity. Such an attempt is made in the next section.

5. Parametric Square Wave Signal

The next idea is that square wave signal can be approached through some other signal using mathematical tools. For $\alpha \in \mathbb{R}$, a parametric control can be defined on quadratic wave signal with $0 < \alpha < 1$. This signal is given by eq. (9).

$$S_\alpha(t) = \begin{cases} 1 & if\ 0 \le t < \frac{1-\alpha}{2}T \\ \frac{1}{\alpha}\left(-\frac{2}{T}t + 1\right) & if\ \frac{1-\alpha}{2}T \le t < \frac{1+\alpha}{2}T \\ -1 & if\ \frac{1+\alpha}{2}T \le t < T \end{cases} \tag{9}$$

and illustrated in the Fig. 2.

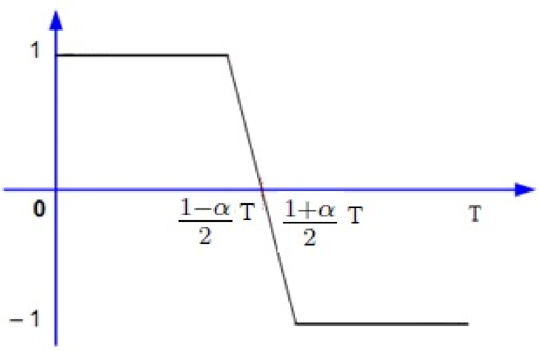

Fig. 2: Parametric quadratic wave signal

Its asymptotic case ($\alpha \to 0$) will approach rectangular wave signal as shown in Fig. 3 [10]. The new signal has piecewise continuous structure. Then it's mean and rms should be the same as discussed in Section 4. Interestingly, its mean is the same but the rms is different (more). Using eq. (1) the mean value of (9) is

$$mean_{\alpha} = (1 - 0.5\alpha)$$

For $\alpha \to 0$ the mean value is

$$mean_{\alpha \to 0} = \lim_{\alpha \to 0}(1 - 0.5\alpha)$$
$$= 1$$

This mean is the same as discussed in section 4. Here both square wave signals have the same mean.

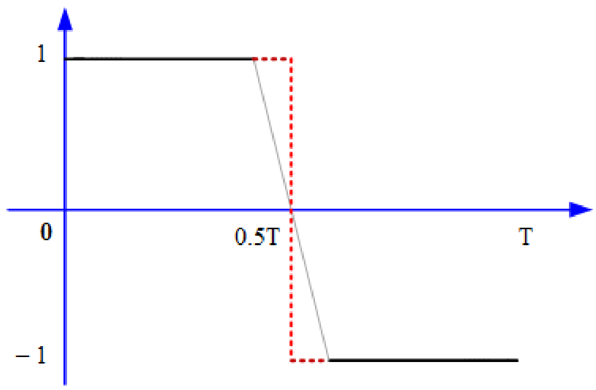

Fig.3: Parametric square wave signal

To calculate its power, we first calculate the Fourier coefficients of eq. (9), which are

$$\begin{cases} a_0 = 0 \\ a_k = 0 \\ b_k = \left(\frac{2}{k\pi} + \frac{2}{\alpha k^2 \pi^2} sin(\alpha k \pi)\right) \end{cases} \tag{10}$$

Using (10) in eq. (5), the power associated with this signal is

$$P_{\alpha} = 0.5 \sum_{k=1}^{\infty} \left(\frac{2}{k\pi} + \frac{2}{\alpha k^2 \pi^2} \sin(\alpha k \pi)\right)^2$$

For $\alpha \to 0$ the power is

$$P_{(\alpha \to 0)} = \lim_{\alpha \to 0}\left(0.5 \sum_{k=1}^{\infty} \left(\frac{2}{k\pi} + \frac{2}{\alpha k^2 \pi^2} \sin(\alpha k \pi)\right)^2\right)$$
$$= 1.3333$$

and the corresponding root mean square value is:

$$rms_{(\alpha \to 0)} = 1.1547$$

The power and root mean square value of square wave signal are more than the values discussed in section 4.

Next error in rms relative to sine wave signal is:

$$error\ \% = \frac{1}{1.1547} \times 11.1107$$
$$= 3.81\%$$

By this control strategy, the error in rms for square wave signal has reduced and is more close to its true value.

6. Conclusions

In this paper a mathematical approach is presented via control theory. Sinusoidal signal is pure AC and it's rms true hence its accuracy specification is applied to all other wave forms. An average responding multimeter measures error in rms for square wave signal as 11.11%. This error can be reduced by approaching square wave signal, by defining a control on quadratic wave signal. Then the signal has piecewise continuous structure and error in rms is 3.81%. Hence this piecewise structure of square wave signal is more suitable as compared to its discrete structure.

References

[1] R. Priemer, Introductory Signal Processing. World Scientific, 1991.

[2] D. Sundararajan, "A Practical Approach to Signals and Systems", John Wiley & Sons (Asia), Pvt. Ltd, 2008.

[3] M. Hulak and M. F. Alves, "On the analysis of (Un) true root mean square measurement, Instituto Superior de Engenharia, 1999.

[4] M. Albu, and G. T. Heydt, "On the *rms* values in power quality assessment". IEEE Transactions on Power Delivery, vol. 18, no. 4, pp. 1586-1587, 2003.

[5] J. Bird, "Engineering Mathematics", 5th Ed., Elsevier Ltd. 2007.

[6] A. Jeffrey, "Advanced Engineering Mathematics", Harcourt/ Academic Press, 2002.

[7] I.S. Gradshteyn and I.M. Ryzhik, "Tables of Integrals, Series and Products", 6th Edn., San Diego, CA: Academic Press, p. 1101, 2000.

[8] H.P. Hsu, "Schaum's outlines of theory and problems of signals and systems", McGraw Hill, 1995.

[9] A.R. Thompson, J.M. Moran, and G.W. Swenson, "Interferometry and synthesis in radio astronomy", New York: Wiley, 2nd Ed., 2001.

[10] B. Ahmad, "More suitable values of Power and Root mean square (RMS) of periodic rectangular wave signal", Int. J. Signal Processing Systems, Engineering and Technology Publishing, vol. 2, no. 2, pp. 128-131, 2014.