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NUMERICAL SOLUTION FOR HYDROMAGNATIC FLUID FLOW BETWEEN TWO HORIZONTAL PLATES, BOTH THE PLATES BEING STRETCHING SHEETS

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Numerical solution for the flow of an incompressible, steady and viscous electrically conducting fluid between two horizontal parallel non-conducting plates, the lower one is a stretching sheet and the upper one is a porous stretching sheet is found. The effects of flow parameters namely M the magnetic parameter, λ the suction parameter and R the Reynolds number have been observed on velocity profiles. Similarity transformations have been used. The resulting ordinary differential equations are solved by using SOR method and Simpson's (1/3) rule. The results have been improved by Richardson extrapolation. The numerical scheme is straightforward, easy to program and very efficient.

Keywords: Hyderomagnetic fluid, Reynolds number, Similarity transformations.

1. Introduction

The flow induced by stretching boundaries is important for metal industries and extrusion processes in plastic [1, 2]. The fluid flow problems about stretching surface have been studied extensively in various topics such as porous medium, MHD flows, heat transfer and non-Newtonian fluids. Sakiadis [3, 4] examined the boundary layer flow on a continuously stretching surface with a constant speed. Crane [5] found an exact solution of two-dimensional Navier-Stokes equation for a stretching plate. Chiam [6] analyzed steady two dimensional oblique stagnation point flow of a viscous fluid. The magnetohyderodynamic flow over a stretching surface has been studied [7-10] for both permeable and impermeable surfaces. Flow of an electrically conducting non- Newtonian fluid past a stretching surface was studied by Able et al. [11] when a uniform magnetic field acts transverse to the surface. Hayat et al. [12] investigated three dimensional flow over a stretching surface in a viscoelastic fluid. Kumaran et al. [13] obtained an exact solution for a boundary layer flow of an electrically conducting fluid past a quadratically stretching and linearly permeable sheet.

This study investigates hyderomagnetic fluid flow between two horizontal plates, both the plates being stretching sheets to extend the numerical work of Dash and Tripathy [14] for ranges of the flow parameters $1 \le M \le 4$, $1 \le \lambda \le 3$ and $0.05 \le R \le 0.8$.The accuracy of numerical results is checked by using three different grid sizes. The results are found in good agreement.

2 Mathematical Analysis

This flow is considered in the presence of a transverse magnetic field. Two equal and opposite forces are introduced to stretch the lower and the upper plates in a way that the position of the points (0, *k*) and (0, -*k*) remains unchanged. Cartesian coordinate system is used where the y-axis is perpendicular to the plates located at y= *k*, y=- *k*. The fluid with constant velocity V_0 is injected through the upper porous plate. The external electric field is zero and the electric field due to polarization of charges is negligible. The induced magnetic field is neglected which is valid for small magnetic Reynolds number.

The governing equations of motion are given below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

where u, v are velocity components and υ is kinematic viscosity coefficient.

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The boundary conditions are:

$u = cx, v = 0$ at y=- k , $c > 0$

$u = cx$, $v = V_0$ at y= k , $c > 0$ (4)

Using similarity transformations:

$$u = cxf'(\eta), \quad v = -ckf(\eta), \eta = \frac{y}{k} \tag{5}$$

where the primes denote differentiation with respect to η and $u = cx$ represents the velocity of both the plates. Equation of continuity (1) is identically satisfied.

Substituting the relations in equation (5) in to equations (2) and (3), we have

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = c^2 x(f'^2 - ff'' - \frac{1}{R}f''' + \frac{M^2}{R}f') \tag{6}$$

$$-\frac{1}{\rho k}\frac{\partial p}{\partial \eta} = c^2 k(ff' + \frac{1}{R}f'') \tag{7}$$

Now, differentiating equation (6) with respect to η and equation (7) with respect to x to get

$$f''' - R(f'^2 - ff'') - M^2 f' = \beta \tag{8}$$

where β is the constant of integration and $M = \sqrt{\sigma/\rho\nu}\, B_0 k$, $\lambda = \frac{V_0}{ck}$ and $R = \frac{ck^2}{\upsilon}$. While σ denotes electrical conductivity, B_0 is strength of transverse magnetic field, ρ is the fluid density.

The corresponding boundary conditions become:

$$f(-1) = 0,\ f'(-1) = 1$$
$$f(1) = \lambda,\ f'(1) = 1 \tag{9}$$

3 Finite Difference Equations

For numerical purpose, let $f' = q$ (10)

Then equation (9) becomes:

$$q'' - R(q^2 - fq') - M^2 q = \beta \tag{11}$$

The equation (11) is discritized by central difference approximation at a typical point $\eta = \eta_n$ of the interval [0,∞) to yield

$$(2 + hRq_n)q_{n+1} - (4 + 2h^2M^2 + 2h^2Rq_n)$$
$$q_n + (2 - hRq_n)q_{n-1} - 2h^2\beta = 0 \tag{12}$$

where h denotes a grid size and equation (10) is integrated numerically. Also the symbols used denote $q_n = q(\eta_n)$. For computational purposes, we shall replace the interval [0,∞) by [0, b], where b is sufficiently large.

The finite difference equation (12) and the first order ordinary differential equations (10) are solved simultaneously by using SOR method, Smith [15, p.262] and Simpson's (1/3) rule, Gerald [16, p.293] with the formula given by Milne [17, p.48] respectively, subject to the appropriate boundary conditions.

The SOR procedure gives the solution of $f' = q$ in the order of accuracy $O(h^2)$ due to second order finite differences used for derivatives involved and Simpson's (1/3) rule gives the order of accuracy $O(h^5)$ in the solution of *f*. Higher order accuracy $O(h^6)$ in the solution of $f' = q$ is achieved by using Richardson's extrapolation, Burden [18, p.168].

4. Results and Discussion

The numerical results have been computed for different values of flow parameters for ranges $1 \le M \le 4$, $1 \le \lambda \le 3$ and $0.05 \le R \le 0.8$. The accuracy of numerical results is checked by using three different grid sizes. The results are found in good agreement. Our numerical technique is straightforward and easy to program.

The effects of the flow parameters have been studied on the primary velocity f' and transverse velocity *f*. It has been noticed that velocity field is almost symmetric about the centre of the channel $(\eta = 0)$ in case of both the plates are being stretched at the same rate but it is not the case with the stretching of the lower plate only. It has been noted that f' increases in the lower half of the channel for increasing R(R<1.0) and decreases, in the upper half of the channel. The tables 1to 2 show that the numerical scheme is also very efficient. The results for f' in the higher order accuracy O(h^6) are given in tables 3 to 6. It is observed that an increase in the value of R increases *f* at all points and transverse velocity increases with increase of η (channel width), when *M* is constant.

The effect of λ on the primary flow f' is maximum at the center of the channel for fixed values of *M*. Also this effect is same, either both the sheets are being stretched or the single sheet is being stretched. Detailed comparison, both tabular and graphical for λ=1, λ=2 and λ=3 shows that the suction parameter radically changes the primary flow velocity f'. But the value of f increases with the increase of λ when M is constant.

When λ is constant and small (λ=1) the Lorentz force decreases the primary flow velocity f' near the lower plate, and increases it near the upper plate. The results have been presented in graphical form in Figures 1 to 3.

Table 1. Optimum value of relaxation parameter ω_{opt} in SOR method when both the plates are being stretching sheets.

M	λ	R	Number of Iterations(NI) in SOR method with ω_{opt}					
			h=0.1		h =0.05		h =0.025	
			NI	ω_{opt}	NI	ω_{opt}	NI	ω_{opt}
1.0	1.0	0.05	31	1.60	68	1.65	220	1.70
3.0	1.0	0.05	30	1.60	38	1.65	67	1.70
3.0	3.0	0.05	75	1.50	79	1.60	92	1.65
3.0	3.0	0.20	75	1.80	79	1.85	140	1.90
2.0	3.0	0.05	37	1.60	162	1.65	394	1.71
4.0	3.0	0.05	33	1.60	38	1.65	48	1.70
4.0	1.0	0.40	33	1.60	35	1.65	49	1.70
2.0	3.0	0.80	39	1.60	47	1.65	82	1.70
4.0	3.0	0.10	33	1.60	38	1.65	54	1.70
4.0	3.0	0.80	43	1.60	52	1.65	59	1.70

Table 2. Optimum value of relaxation parameter in SOR method when the lower plate being stretching sheet.

M	λ	R	Number of Iterations(NI) in SOR method with ω_{opt}					
			h =0.1		h =0.05		h =0.025	
			NI	ω_{opt}	NI	ω_{opt}	NI	ω_{opt}
1.0	1.0	0.20	23	1.60	56	1.65	160	1.70
3.0	1.0	0.20	27	1.10	84	1.15	263	1.20
1.0	3.0	0.20	40	1.60	45	1.65	140	1.74
1.0	1.0	0.25	28	1.40	97	1.5	289	1.60
1.0	1.0	0.30	37	1.70	70	1.80	130	1.85
3.0	1.0	0.25	31	1.60	35	1.65	58	1.70

Table 3. M=1.0, $\lambda = 1.0$, R=0.05 M =3.0, λ =1.0, R = 0.05.

Numerical Results using Richardson Extrapolation Method									
h=0.2	h =0.1	h =0.05	Extrapolated		h =0.2	h =0.1	h =0.05	Extrapolated	
η	f'	f'	f'	f'	η	f'	f'	f'	f'
0.000	1.000000	1.000000	1.000000	1.000000	0.000	1.000000	1.000000	1.000000	1.000000
0.400	0.512934	0.512044	0.511817	0.511742	0.400	0.485853	0.482648	0.481824	0.481548
0.800	0.289751	0.288500	0.288182	0.288076	0.800	0.342666	0.340097	0.339443	0.339223
1.200	0.291790	0.290538	0.290220	.290114	1.200	0.343433	0.340849	0.340190	0.339970
1.600	0.517144	0.516253	0.516027	0.515952	1.600	0.487916	0.484681	0.483850	0.483572
2.000	1.000000	1.000000	1.000000	1.000000	2.000	1.000000	1.000000	1.000000	1.000000

Table 4. M =1.0, λ = 3.0, R =0.20 M = 2.0, λ = 1.0, R = 0.05.

Numerical Results using Richardson Extrapolation Method									
h =0.2	h =0.1	h =0.05	Extrapolated		h =0.2	h =0.1	h =0.05	Extrapolated	
η	f'	f'	f'	f'	η	f'	f'	f'	f'
0.000	1.000000	1.000000	1.000000	1.000000	0.000	1.000000	1.000000	1.000000	1.000000
0.400	1.789670	1.790962	1.791286	1.791395	0.400	0.767668	0.767608	0.767591	0.767586
0.800	2.030021	2.031768	2.032206	2.032352	0.800	0.585113	0.585109	0.585107	0.585105
1.200	1.811893	1.813531	1.813942	1.814079	1.200	0.416443	0.416513	0.416529	0.416534
1.600	1.150124	1.151166	1.151429	1.151516	1.600	0.231219	0.231311	0.231334	0.231341
2.000	0.000000	0.000000	0.000000	0.000000	2.000	0.000000	0.000000	0.000000	0.000000

Table 5. M =4.0, λ =1.0, R =0.05 M =2.0, λ =1.0, R =0.05 .

Numerical Results using Richardson Extrapolation Method									
h =0.2	h =0.1	h =0.05	Extrapolated		h =0.2	h =0.1	h =0.05	Extrapolated	
η	f'	f'	f'	f'	η	f'	f'	f'	f'
0.000	1.000000	1.000000	1.000000	1.000000	0.000	1.000000	1.000000	1.000000	1.000000
0.400	0.473828	0.469650	0.468560	0.468193	0.400	1.527394	1.531557	1.532644	1.533010
0.800	0.368448	0.366258	0.365699	0.365511	0.800	1.632242	1.634416	1.634972	1.635159
1.200	0.368855	0.366644	0.366079	0.365890	1.200	1.631037	1.633260	1.633830	1.634021
1.600	0.475190	0.470967	0.469864	0.469493	1.600	1.523363	1.527618	1.528729	1.529104
2.000	1.000000	1.000000	1.000000	1.000000	2.000	1.000000	1.000000	1.000000	1.000000

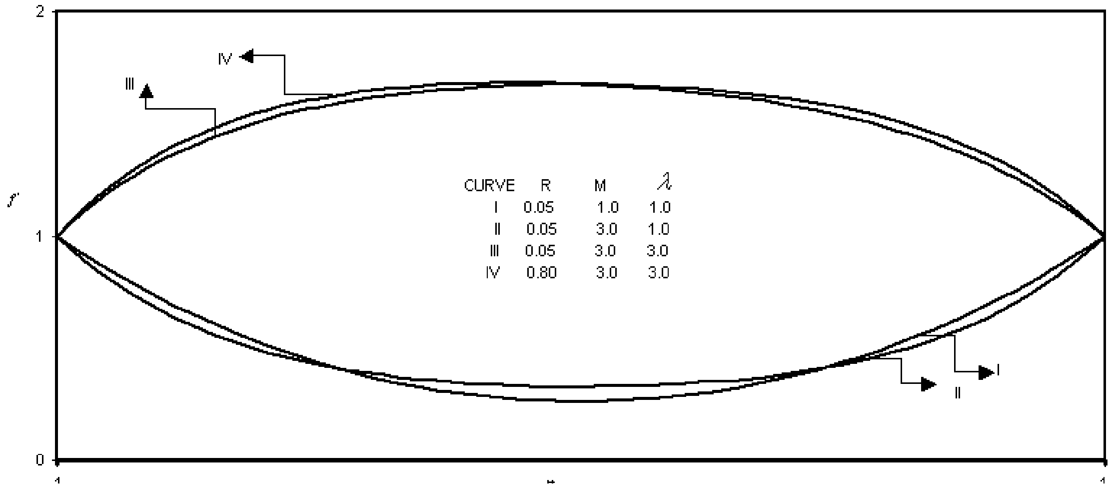

Figure 1. Graph of f' when both the plates are stretching sheets.

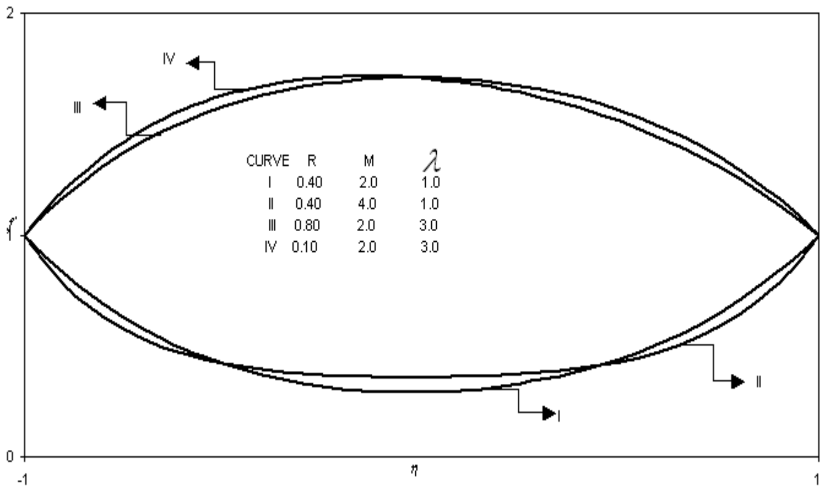

Figure 2. Graph of f' when both the plates are stretching sheets.

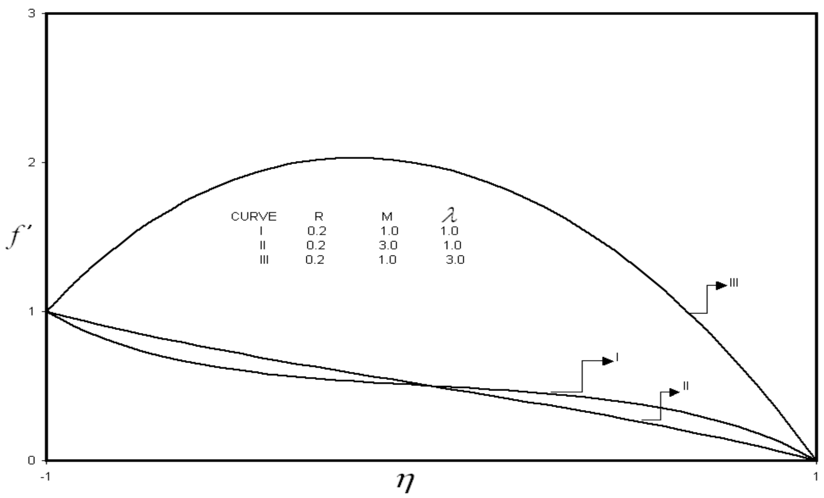

Figure 3. Graph of f' when lower plate being a stretching sheet.

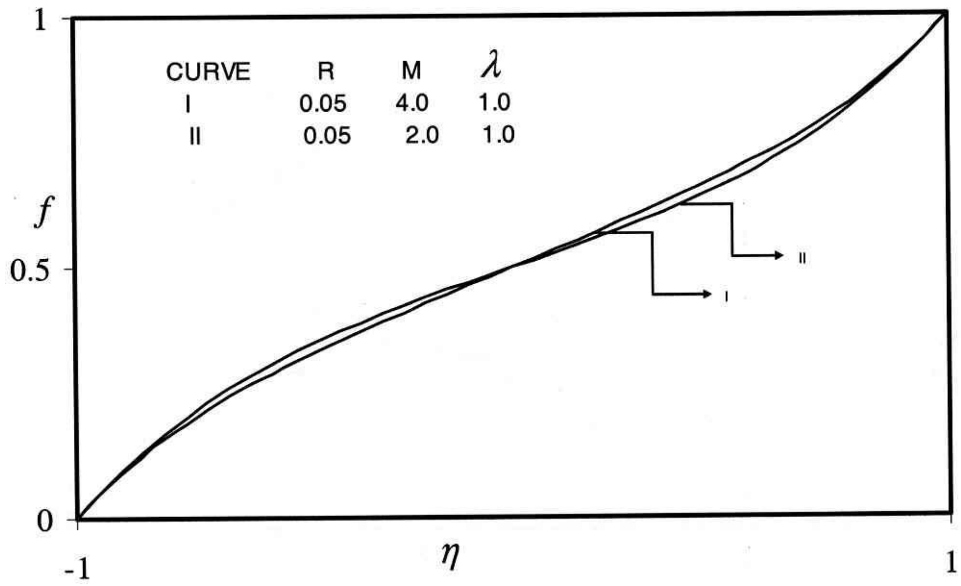

Figure 4. Graph of *f* for different values of *M*, *R* and λ.

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