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PREDICTION OF SOLAR CYCLE 24 USING FOURIER SERIES ANALYSIS

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Predicting the behavior of solar activity has become very significant. It is due to its influence on Earth and the surrounding environment. Apt predictions of the amplitude and timing of the next solar cycle will aid in the estimation of the several results of Space Weather. In the past, many prediction procedures have been used and have been successful to various degrees in the field of solar activity forecast. In this study, Solar cycle 24 is forecasted by the Fourier series method. Comparative analysis has been made by auto regressive integrated moving averages method. From sources, January 2008 was the minimum preceding solar cycle 24, the amplitude and shape of solar cycle 24 is approximate on monthly number of sunspots. This forecast framework approximates a mean solar cycle 24, with the maximum appearing during May 2014 (± 8 months), with most sunspot of 98 ± 10. Solar cycle 24 will be ending in June 2020 (± 7 months). The difference between two consecutive peak values of solar cycles (i.e. solar cycle 23 & 24) is 165 months[± 6 months].

Keywords: Solar cycle 24, Fourier series, Consecutive peak, Maximum number of sunspots

1. Introduction

For solar cycle activity prediction timing and magnitude is an important problem in the field of science and technology. When an active solar period occurs, several robust eruptions take place on the Sun which increases gradually, such as coronal mass ejections, flares, high velocity solar wind photons, that lead to adverse effect on the Earth's ionosphere and the geomagnetic field, with effects on the low atmosphere. Hence it becomes essential to produce a consistent method of forecasting solar cycle and the incoming solar activity.

The recently developed solar cycle 24 seems strange from several viewpoints, due to its global low activity and uncommonly extended minimum period as compared to the previous three or four cycles. At present, several predictions on the incidence of the solar cycle 24 maximum, using different methods, both precursor and non-precursor. We used Fourier Series method to forecast length and amplitude of Solar cycle 24. These predictions are also compared with ARIMA model.

This study presents an analysis of the problem of predicting sunspots. In section 2, brief literature review is presented, section 3 lists data description and research methods used, section 4 extends the discussion and the results and the conclusion is drawn in section 5

2. Literature Review

The length of each solar cycle is about 11 years from one minimum to the following and is given a number, with SC 1 beginning around 1755 [2]. The SC 23 has continued longer than expected and at present, the minimum of SC 24 is still not defined precisely. The Solar and Heliospheric Observatory (SOHO), in the first week of January 2008, displayed reversed polarity high latitude sunspot. Some specialists went as far as saying that this event is the beginning of SC24 [8]. Nevertheless, sunspots with the same magnetic polarity as sunspots for SC 23 were shown in March 2008. According to the NASA solar physicist, SC 24 has started, however two solar cycles are concurrently in progress until cycle 24 raises its activities. These observations are a sign that the SC 24 maximum may probably not occur until 2012 [6]. On the basis of the above information, January 2008 is marked as the start of SC 24. Several predictions on the solar cycle 24 have been attempted. Pesnell [3] details 45 predictions of SC 24 by various methods as put forward by different researchers. Looking at these predictions, variations are clear both in the timing and magnitude of SC24. While experts forecast a lower activity cycle with a value of 42±35 for the SSN maximum, others predict a much higher solar activity for SC 24 reaching a SSN maximum of 180±32. The report specifies that approximately half of the predictions agree that SC 24 will be an usual cycle within the range of the achieved SSN maximum for SC 23. Pesnell [3] in spite of the noticeable differences observed in the predictions, focuses on the need of having quantitative approximations of the uncertainty of the predictions, which will help us in estimating the probability results of space weather in Solar cycle 24.

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3. Data and Methodology

The monthly data used in this research was taken from 1749 to 2008. According to Kane [1] "The quality of the data is considered as poor during 1700–1748, questionable during 1749–1817, good during 1818–1847, and reliable since 1848". The greatest limit of intensity of each cycle and the time from the starting of the cycle to its maximum (hereafter "ascension time") was drawn from cycles 1 (1755 February) to 23 (2008 January). The data put to use during the study were acquired from the National Aeronautics and Space Administration [7]. To make our predictions more accurate, we used two forecasting methods on this data. One of them is the Fourier series method. This method is well documented in Bloomfield [4]. Fourier analysis deals with exploration of cyclical patterns of data. The aim was to decompose a complex time series with cyclical components into a few underlying sinusoidal functions of particular wavelengths.

The Fourier Series model is said to have the following form:

$$Y_F = a_o + \sum_{i=1}^{n} a_i Cos(2\pi tf) + \sum_{i=1}^{n} b_i Sin(2\pi tf) \quad (1)$$

$$a_i = \frac{2}{m} \sum_{t=1}^{m} y_t \, Cos(2\pi tf) \quad (2)$$

$$b_i = \frac{2}{m} \sum_{t=1}^{m} y_t \, Sin(2\pi tf) \quad (3)$$

Where Y_F is the simulated Fourier Series of the Sunspot cycle, n designates the number of peaks in the periodogram (in the frequency domain), m is the length of the original time series, y_t is the original time series of the Sunspot cycle, f is the Fourier frequency of the corresponding peak in the periodogram, t is the time, a_o is the mean value of y_t.

The second method which is applied here is ARIMA. The ARIMA method was presented by Box & Jenkins [5]. The Box-Jenkins variant of the ARMA model is frequently used for everyday applications of non-stationary time series which later offer differencing convert to stationary. The globally renowned method is ARIMA (p, q, d), in which p gives the amount of autoregressive parameters, q is the number of moving averages parameters, and d represents differencing passes. The selections will be made by observing the Auto Correlation function and Partial Auto Correlation function. After p and q have been chosen and defined, there will take place a process of intercept and coefficient estimation. The mathematical formulation of the ARIMA (*p, d, q*) model is given in the following terms.

$$\left(1 - \sum_{i=1}^{p} \phi_i L^i\right)(1-L)^d \; Y_t = \left(1 + \sum_{j=1}^{q} \theta_j L^j\right)\varepsilon_t \quad (4)$$

where ε_t gives the random error. The backshift operator is defined as $L^i y_t = y_{t-i}$ with ϕ_i is the autoregressive coefficient of $\log(i)$, θ_i is the moving average coefficient of $\log(i)$. The expansion of the term $(1-L)^d$ in this model, has been done by general binomial theorem.

4. Result and Discussion

In this paper, ARIMA and Fourier analysis is not directly applied to Sunspot monthly data. Duration of each Solar cycle is calculated, then, graph is drawn between Solar cycle 1 and its duration, when ARIMA and Fourier series analysis is applied on it, the forecasted duration of Solar cycle 24 is obtained. To obtain a peak value of Solar cycle, graph showing Solar cycle on x-axis and peak values of Solar cycle on y-axis has been constructed. Then the forecasted peak value of Solar cycle 24 has been obtained by applying ARIMA and Fourier analysis. Similarly, the forecasted maximum number of Sunspot has been achieved.

Predictions using ARIMA (3,0,2) for solar cycle 24 shows that the maxima (highest number of sunspots) is expected in March 2012 (± 6 months). Predictions designate that after 139 months March 2012 will prove to be the month with the greatest number of Sunspots number with the range of 118 (± 8 Sunspot numbers) by using ARIMA (4,1,4). The deviation Solar cycle 24 consists of 82 (± 6 months) which will end in October 2015 [±6 months]. We obtain this value by using ARIMA (2,0,2) (Figure 1)

Table 1. Analysis of both models (ARIMA and Fourier Series).

Model	Expected Number of Sunspot	Peak Time	Ending Time
ARIMA	118	March 2012	October 2015
Fourier Series	98	May 2014	June 2020

The Fourier Series (8) for Solar cycle 24 calculates that the peak of monthly Sunspot (estimated value 98±10) will occur in May 2014, after 165 months of previous peak. Date of end of the Solar cycle 24 has been forecasted to be June 2020[±7 months] using Fourier Series (7) (see Fig 2).

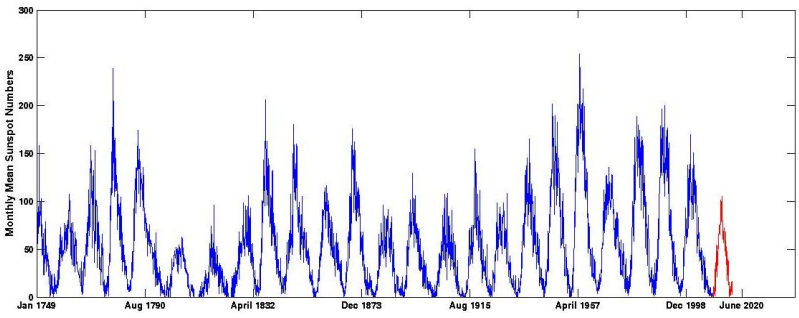

Figure 1. Red lines show forecasted Solar cycle 24 by ARIMA. This cycle will be completed in October 2015.

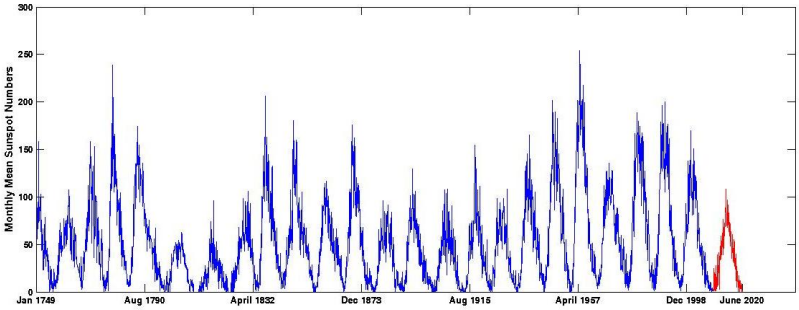

Figure 2. Red lines show forecasted solar cycle 24 by Fourier Series(8). This cycle will be completed in June 2020.

5. Conclusion

By means of the two methods used here, the Fourier series Analysis and ARIMA, we were able to conclude from the significant trends from the data of the International sunspot numbers, the cycle duration, the peak of the cycle and its maximum intensity. Our prediction results are underlaid by the framework that was checked out by sample in order to have high accuracy.

In this research, we discovered that Fourier Series analysis covers sunspot data more precisely than ARIMA. Above forecasting framework approximately a mean solar cycle 24 with the most appearing during May 2014 (± 8 month), hence the number predicted to be 98 ± 10. Solar cycle 24 will be completed in June 2020 (± 7 months). Two consecutive peak values of solar cycles have a difference of 165 months (± 6 months). Historical data show that there are no more than three consecutive strong solar cycles. Prediction of Solar cycle 24 also confirmed that this cycle is weak as compared to previous three solar cycles.

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