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FORMATION OF LOPSIDED AND BAR STRUCTURES IN NON-STATIONARY GRAVITATING SYSTEMS. I- ISOTROPIC CASE

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This is an examination of the gravitational instability of bar-like and lopsided structures of non-stationary isotropic disk model. Non-stationary analogs of the dispersion equation of these two oscillation modes are discussed, in this paper. Growth rates of both oscillation modes are found with the help of non-stationary dispersion equation. Results are presented in the form of graphs which show dependence of initial virial ratio on rotation parameter Ω. A comparative analysis of the growth rates of both oscillation modes is made and found that lopsided structures overall dominates the bar-like structures.

Keywords: Non-stationary, Asymmetric oscillation, Potential perturbation, Virial ratio, Critical value, Gravitating system, Rotation parameter

1. Introduction

Most spiral galaxies in the Universe have a bar structure in their centre. Galaxies' star-filled bars are thought to emerge as gravitational density waves funnel gas toward the galactic centre, supplying the material to create new stars [1]. Some astronomers have suggested that the formation of a central bar-like structure might signal a spiral galaxy's passage from intense star-formation into adulthood, as the bars turn up more often in galaxies full of older, red stars than younger, blue stars. This storyline would also account for the observation that in the early Universe, only around one fifth of spiral galaxies contained bars, while more than two thirds do in the more modern cosmos [2]. Lopsidedness in galaxies is defined as "If it displays a global non-axis symmetric spatial distribution of type $m=1$; where m is the azimuthal wave number or a $\cos\phi$ distribution, where ϕ is the azimuthal angle in the plane" [11]. By observations, lopsidedness or asymmetric, instead of bisymmetry, has been found in a large number of galaxies on both small and large scales [9].

Many authors have put forward various specific models of the self-gravitating systems. Binney and Tremaine [13] have assembled a large number of results .The basis of the most of these results is on the linearization of the Euler-Poison and Vlasov Poison systems around a stationary solution. Kalnajs [3] has covered milestones in stationary models of self-gravitating systems. Although the stationary models of gravitating systems are abundance in the research, the presence of non-stationary models is very conspicuous among various models for study of dynamical development of large-scale structures.

In this paper, we compare the critical dependence of initial virial ratio on rotation parameter Ω for non-stationary isotropic disk model, for bar-like and lopsided oscillation modes and will find that which structure is more unstable than other.

2. Initial Non-stationary Isotropic Model

Nuritdinov [5, 6] constructed non-stationary non-linear isotropic model for disk-like self-gravitating systems. The phase density of this non-stationary model is given by

$$\Psi\ r, v_r, v_\perp, t\ = \frac{\sigma_o}{2\pi\Pi\sqrt{1-\Omega^2}} \left[\frac{1-\Omega^2}{\Pi^2}\left(1-\frac{r^2}{\Pi^2}\right) - \ v_r - v_a\ ^2 - \ v_\perp - v_b\ ^2\right]^{-\frac{1}{2}} \cdot \chi\ R-r \qquad (1)$$

with

$$\Pi\ t\ = \frac{1+\lambda\cos\psi}{1-\lambda^2} \qquad (2)$$

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$$t = \frac{\psi + \lambda \sin\psi}{(1-\lambda^2)^{3/2}} \qquad (3)$$

Where Ω is a dimensionless parameter which describes the rotation of disk with value $0 \le \Omega \le 1$. The parameter $\lambda = 1 - \left(\frac{2T}{|U|}\right)_o$ is virial ratio of the gravitating system. At $t = 0$, that is at $\lambda = 0$, we get an equilibrium disk of Kalnaj [3]. The function $\Pi(t)$ has a sense of tension coefficient of the system.

The radial and tangential components of velocity of the "particle", in the non-stationary model (1), v_r and $v_\perp$, with coordinates $\vec{r} = (x, y)$, the modulus of this is expressed with corresponding equilibrium coordinate r_o in the form $r = \Pi(t)\, r_o$. Also, in equation (1),

$$V_a = -\lambda\sqrt{1-\lambda^2}\,\frac{r \sin\psi}{\Pi^2} \qquad (4)$$

$$v_b = \frac{\Omega r}{\Pi^2} \qquad (5)$$

By considering a small perturbation on the background of non-stationary isotropic model (1), the following equation of motion for the displacement vector of centroid $\delta \mathrm{r}$ is obtained; which is

$$\Lambda \delta\vec{r} = \Pi^3(\psi)\,\frac{\partial(\delta\Phi)}{\partial\vec{r}} \qquad (6)$$

$\delta\Phi$ is potential perturbation taken at the point (x, y, z). Because the deviation in path of a particle that lies in perturbed state at current time depends on the field state at previous times, $\psi_1 \in (-\infty, \psi)$ and our aim is to look for an instability, it is safe to presume that $\psi_1 = -\infty$, $\Delta x = \Delta y = 0$. At the current time at each point, these are particles that possess different velocities, so that the calculation of perturbation in the density or deformation of the boundary of the system in question it is necessary to continue to the displacement of the centroid $\overline{\delta x}, \overline{\delta y}$ taking aggregate of the equation (6) taking velocity space into account. The operator Λ in equation (6) is defined as

$$\Lambda = (1 + \lambda\cos\psi)\left(\frac{d^2}{d\psi^2} + \lambda\sin\psi\,\frac{d}{d\psi} + 1\right) \qquad (7)$$

Due to the fact that model (1) is non-stationary and non-linearly, the analysis of its stability becomes rather tougher than for the corresponding equilibrium disk, as it is relatively harder to derive Non-stationary analogue of the dispersion relation (NADR) in general case. (Remember that the corresponding equilibrium model's stability has been focused on, agreeably, by quite a few authors [3, 12]. We already found Non-stationary dispersion relation of these two modes, which we described in [8, 9,11]. For lopsided structures, the non-stationary dispersion relation is given by

$$\Lambda L_\tau(\psi) = \frac{3}{8(1+\lambda\cos\psi)^4} K(\psi)\,(\lambda + \cos\psi)^{2-\tau}\sin^\tau\psi \quad ; \quad \tau = 0 - 2 \qquad (8)$$

where

$$K(\psi) = \begin{bmatrix} 11(\lambda+\cos\psi)^2 + (5\Omega^2 - 4)(1-\lambda^2)\sin^2\psi \\ -10i\Omega\sqrt{1-\lambda^2}(\lambda+\cos\psi)\sin\psi \end{bmatrix} L_o(\psi) + \begin{bmatrix} 10(3-\Omega^2)(1-\lambda^2)(\lambda+\cos\psi)\sin\psi \\ +10i\Omega\sqrt{1-\lambda^2}\left[(\lambda+\cos\psi)^2 - (1-\lambda^2)\sin^2\psi\right] \end{bmatrix} L_1(\psi) + \begin{bmatrix} 11(1-\lambda^2)^2\sin^2\psi + (5\Omega^2 - 4)(1-\lambda^2)(\lambda+\cos\psi)^2 \\ +10i\Omega(1-\lambda^2)^{3/2}(\lambda+\cos\psi)\sin\psi \end{bmatrix} L_2(\psi) \qquad (9)$$

The Non-stationary Dispersion Relation for bar-like structures is of the form:

$$\Lambda L_\tau(\psi) = \frac{3(\lambda+\cos\psi)^{1-\tau}(\sin\psi)^\tau}{2(1+\lambda\cos\psi)^2} B(\psi) \quad ; \quad \tau = 0 - 1 \qquad (10)$$

where

$$B(\psi) = \left(\cos\psi + \lambda - i\Omega\sin\psi\sqrt{1-\lambda^2}\right)\left(L_o(\psi) + QL_1(\psi)\right) \qquad (11)$$

We can calculate the corresponding growth rates using the formula :

$$I_{nc} = \frac{(\|K_{\max}\|)}{P(\lambda)} \qquad (12)$$

Graphs of Lopsided and Bar Mode: Isotropic Case

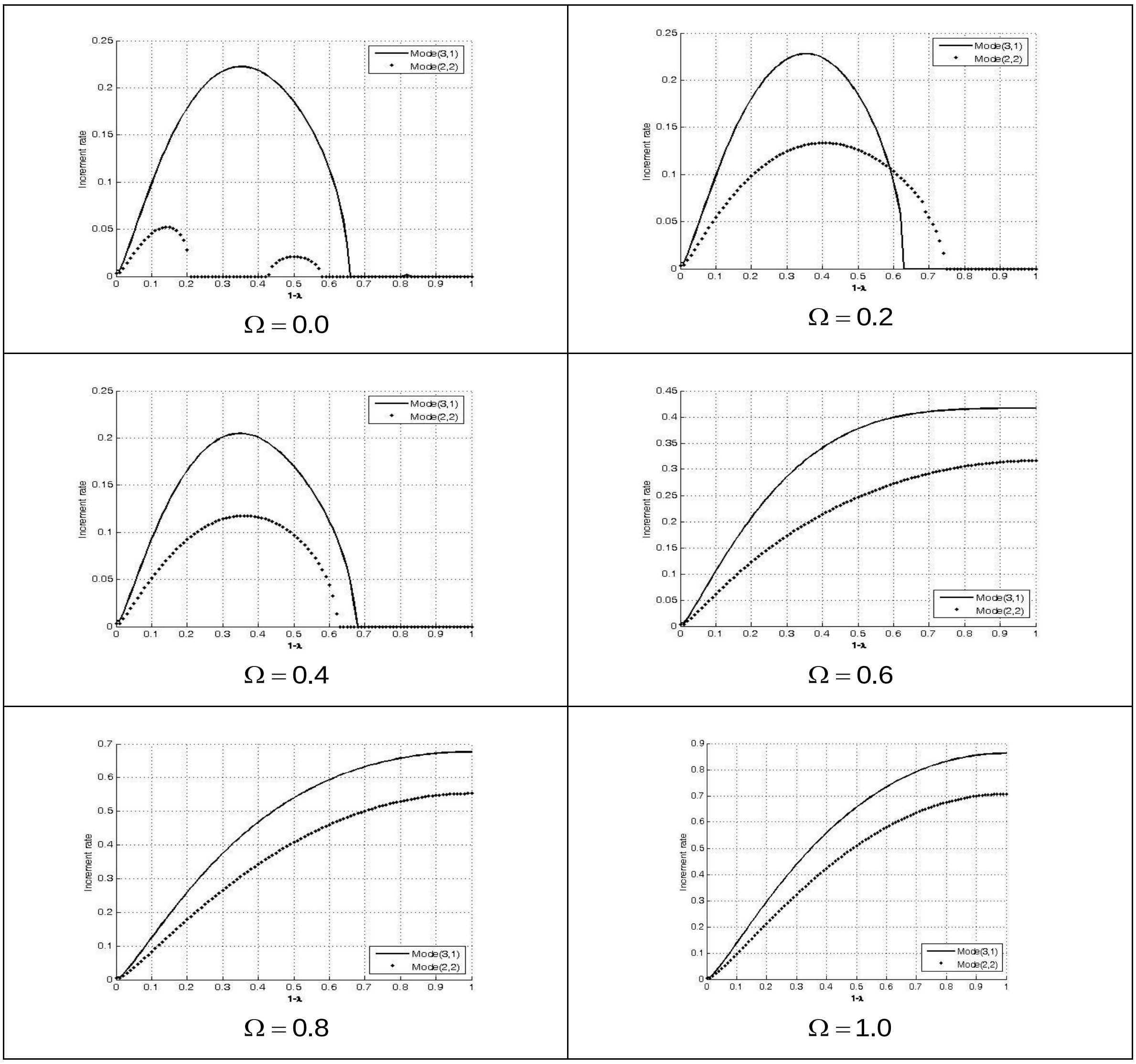

Figure 1. Graphs showing critical dependence of initial virial ratio on rotation parameter for Lopsided and Bar-like oscillation modes.

Where $\|K_{max}\|$ is the natural logarithm of the largest value of the modulus of the roots of characteristic equation and $P(\lambda)$ is the period of radial oscillation which is given by

$$P(\lambda) = \frac{2\pi}{(1-\lambda^2)^{3/2}} \qquad (13)$$

With the help of these dispersion relations (Equation 8,10) and the formula of growth rates (Equation 12,13), we can construct the critical diagram of initial virial ratio and rotation parameters at different values of Ω, which give us information about the instabilities of these two oscillation modes. The characteristics roots of Equation (8,10) have been obtained by the method of periodic solution [10].

It is interesting to compare the growth rate of lopsided mode with bar-like mode as these modes inform us about characteristic times for the appearance of these gravitational instabilities. Figure 1 shows the critical dependence of initial virial ratio on rotation parameter for lopsided and bar-like oscillation modes. It is very clear from comparison that the main results of the lopsided mode are always leading on bar-like mode. At all values of Ω, lopsided mode is dominant except at

Ω=0.2with virial ratio=0.6 where instabilities in both modes are equal. This implies that the displacement of the nucleus takes place relative to the center of the system, in advance than bar-like oscillation mode, regardless the value of rotation parameter in an isotropic mode.

3. Conclusion

In this research paper, comparative analysis of two oscillation perturbations modes has been made. Results show that lopsided structure is dominant overall on bar-like structures which shows that the formation of lopsided structures in the self-gravitating system has greater chance than the bar-like structure.

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