

# OPTICAL TAMM STATES AT INTERFACES OF DIFFERENT PERIODIC MEDIA CONTAINING SINGLE AND DOUBLE NEGATIVE MATERIAL LAYERS

MARYAM SAEED and \*MUNAZZA Z. ALI

Department of Physics, University of the Punjab, Lahore, Pakistan

(Received June 10, 2013 and accepted in revised form September 16, 2013)

---

Optical Tamm States (OTS) are investigated at the interfaces of semi-infinite metal and finite one dimensional periodic structures. Three types of periodic structures are considered here. Initially a one dimensional structure of alternate regular material layers is considered. The other two structures consist of alternate left-handed material and regular material layers and alternate single negative layers. The transfer matrix approach is used to derive the dispersion relation for the TE mode Optical Tamm States (OTS). The dispersion relations is plotted in the photonic band gap frequency range and analyzed by changing the structure parameters.

**Keywords:** Metamaterials, Periodic structures, Surface states

---

## 1. Introduction

Optical Tamm states (OTS), a type of surface states, are confined at the interface of a homogenous medium and a periodic structure. Generally, the homogenous medium is a metal below the plasma frequency. These states are localized at the interface and decay on both sides of the interface. The decay of OTS in the metal is due to its negative dielectric constant. However, the decay of field in the periodic multilayer structure is due to photonic band gaps. It has been investigated that OTS can exist both in TE and TM polarizations, at the interface of a metal and a periodic structure of regular materials [1]. It has been observed that the order of layers in the periodic multilayer structure plays a vital role for the existence of OTS and a confined mode of OTS can only exist if the layer of high refractive index is placed adjacent to the metal. The dispersion curve of OTS lies within the light cone; therefore, the direct optical excitation of OTS can be made possible [2].

Optical Tamm states have attracted much interest due to their potential applications. These states are used in polariton laser [3, 4]. Polariton laser is a device which is based on the stimulating scattering of polariton [5]. There is no need of population inversion in the polariton laser as compared to conventional laser. So, a less energy is required to run the polariton laser. This property makes this laser a very useful device in low power

optical telecommunication and light amplification [6]. OTS is also used in resonant optical filters [7], optical diodes [8] and optical absorbers [9]. Recently, an efficient way is introduced for the existence of OTS at the interface of a photonic crystal heterostructure [10].

In this paper, we investigate the existence of OTS at the interface separating a semi-infinite metal and a one dimensional (1-D) periodic multilayer structure. The dispersion relations of OTS are derived for the case of TE mode. Then, the dispersion relation for TE-mode is plotted and analyzed by changing the structure parameters i.e. angle of incidence, width of layers and number of layers.

## 2. Optical Structures and Methods of Calculations

To study the propagation of OTS, let us consider a structure that consists of an interface between a semi-infinite metal and a finite one-dimensional periodic structure. A homogenous medium, that is, a metal (below the plasma frequency) with negative electric permittivity i.e.  $\epsilon = -n^2$  is placed on the left side of the interface ( $z < 0$ ). For ( $z > 0$ ), we have a one-dimensional periodic structure consisting of A and B layers.

Three cases have been considered here, in which different periodic multilayer structures are placed on the right side of the interface. Initially, a one dimensional periodic structure of regular

---

\* Corresponding author : munazzazulfiqar@yahoo.com

material layers is considered. This kind of structure gives rise to Bragg gap [7]. Here a structure is considered with parameters of layer A and layer B given as:

$$\epsilon_A = 1.5, \quad \mu_A = 1, \quad \epsilon_B = 3.0, \quad \mu_B = 1$$

Secondly, we have considered the case in which the periodic structure consists of alternating layers of left-handed (LHM) and right-handed materials (RHM). LHM are artificial composite materials with simultaneous negative values of electric permittivity and magnetic permeability [12, 13]. These materials are also known as double negative materials. A periodic structure consisting of alternate LHM and RHM layers gives rise to a zero-n gap [14]. These materials are inherently dispersive and lossy. Here, for simplicity losses in these materials are neglected. The parameters for the structure considered here are given as:

$$\epsilon_A = 2, \quad \mu_A = 1, \quad \epsilon_B(\omega) = 1 - \frac{100}{\omega^2}, \quad \mu_B(\omega) = 1 - \frac{100}{\omega^2}$$

Thirdly, the periodic structure is considered to be composed of single negative (SNG) materials. Single negative materials are those materials with either negative value of electric permittivity (ENG) or magnetic permeability (MNG). This kind of structure gives rise to zero-phi gap [15]. The parameters considered here are:

$$\epsilon_A = 1 - \frac{100}{\omega^2}, \quad \mu_A = 1, \quad \epsilon_B = 1, \quad \mu_B(\omega) = 1 - \frac{100}{\omega^2}$$

The schematic diagram of an interface separating a semi-infinite metal and a finite I-D periodic multilayer structure is shown in Figure 1. The direction of periodicity is taken along the z-axis and d is the period of the structure, which can be defined as;

$$d = d_a + d_b$$

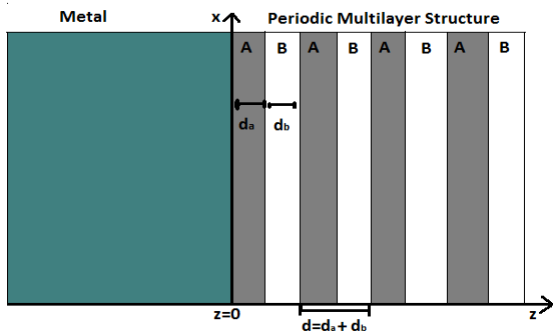


Figure 1. The schematic diagram of the structure under consideration.

We consider surface states localized at the interface, located at  $z=0$ . For the geometry considered here, for TE mode, the electric field is aligned along the y-axis. Using the transfer matrix approach [16], the tangential components of electric and magnetic field at the beginning and end of the  $j^{\text{th}}$  layer are related as:

$$\begin{bmatrix} E(z) \\ H(z) \end{bmatrix}_{z=d} = T_j \begin{bmatrix} E(z) \\ H(z) \end{bmatrix}_{z=0} \quad (1)$$

Where  $T_j$  is the transfer matrix across  $j^{\text{th}}$  layer and can be written as

$$T_j = \begin{bmatrix} \cos(k_j d_j) & \frac{\mu_j}{k_j} \sin(k_j d_j) \\ -\frac{k_j}{\mu_j} \sin(k_j d_j) & \cos(k_j d_j) \end{bmatrix} \quad (2)$$

where  $k_j$  is the wave vector in the  $j^{\text{th}}$  layer and is given by:

$$k_j = \frac{w}{c} n_j \sqrt{1 - \frac{n_m^2 \sin^2 \theta}{n_j^2}} \quad (3)$$

In the above relations,  $n_j$  and  $n_m$  represent the refractive index of  $j^{\text{th}}$  layer and metal respectively. The dispersion relation of OTS for TE-mode can be derived by using the transfer matrix approach as [1]:

$$i \left( \frac{c^2}{w^2} k_x^2 - n_m^2 \right)^{1/2} = \frac{t_{11} - \lambda}{t_{12}} \quad (4)$$

Here  $t_{11}, t_{12}$  are the elements of transfer matrix of the whole structure and  $\lambda$  is the Eigen value of the transfer matrix and can be written as:

$$\lambda = \frac{(t_{11} + t_{12})^2}{2} \pm \sqrt{\frac{(t_{11} + t_{12})^2}{2} - 1} \quad [1].$$

### 3. Results and Discussion

The localized states of OTS existed in the photonic gap region of the periodic structure. The photonic gap may be a Bragg gap, a zero-n gap or a zero-phi gap depending on the type of the periodic multilayer structure. The dispersion relation of OTS for TE-mode, given in equation (4), is plotted in the photonic gap frequency range. The dispersion curves in the Bragg gap, zero-n gap and zero phi gap are shown in the Figure 2(a,b,c) respectively. These curves correspond to the real values of the wave vector  $K_x$  and

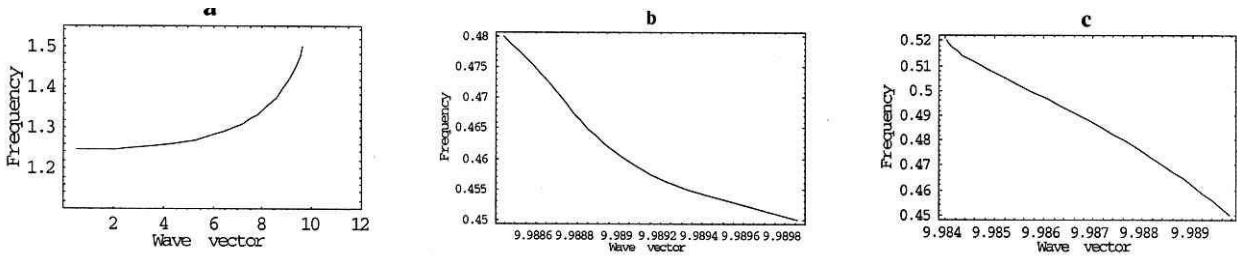


Figure 2. Dispersion plots for TE-mode with plasma frequency of metal  $W_p=10$ . (a) in Bragg gap (b) in zero-n gap (c) in zero-phi gap.

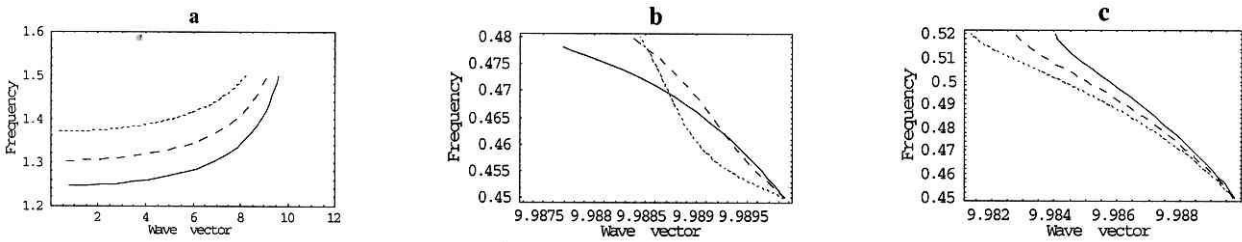


Figure 3. Analysis of dispersion plot at different angle of incidence.

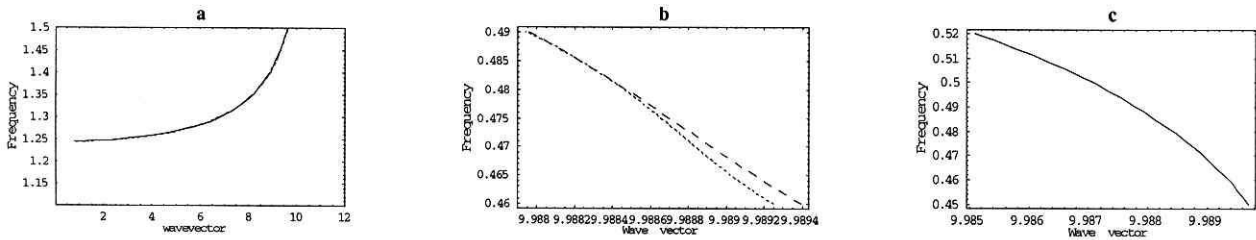


Figure 4. Analysis of dispersion plot at different width of layers.

physically describe the propagation of OTS along the interface. In our computational work, dimensionless units are used. The dimensionless quantities are: the frequency:  $W = \frac{\omega d}{c}$ , the wave

vector:  $K = kd$ , the width of layers  $D_j = \frac{d_j}{d}$  ( $j=A,B$ ).

The angular dependence of the dispersion relation for the three cases considered above is shown in Figure 3. The a, b and c part of Figure 3 correspond to the Bragg gap the zero-n gap and the zero-phi gap respectively. In these figures, the solid, dashed and dotted lines correspond to angles of incidence of  $\pi/6$ ,  $\pi/4$  and  $\pi/3$  respectively. It is observed that the dispersion curves are sensitive to the angle of incidence.

The dependence of these dispersion curves on the width of layers is plotted in Figure 4. For the dispersion curves in the Bragg gap (Figure 4a), the solid, dashed and the dotted lines correspond to the width of layers taken to be

$D_A = D_B = 0.5, D_A = 0.4; D_B = 0.6$  &  $D_A = 0.3; D_B = 0.7$  respectively. Similarly, for the dispersion curves in the zero-n gap (Figure 4b) and the zero-phi gap (Fig.4c), the solid, dashed and dotted lines correspond to the width of layers taken to be  $D_A = 0.4; D_B = 0.6, D_A = 0.3; D_B = 0.7$  &  $D_A = 0.2; D_B = 0.8$  respectively. All these plots clearly show that the OTS in Bragg gap and zero phi gaps are completely insensitive to the width of layers whereas in zero-n gap these states are relatively sensitive to the change in layer widths.

Dispersion properties of OTS can also be studied by changing the number of layers in the periodic structures. Again the a, b and c part of Figure 5 correspond to the Bragg gap, the zero-n gap and the zero-phi gap respectively. The solid, dashed and dotted lines in Figure 5 correspond to 10, 15 and 20 layers respectively. It is observed that dispersion curves in the zero-phi gap and zero-n gap are sensitive to the number of layers as compared to the curve in the Bragg gap. However, it is shown that in zero-n gap the curves diverge

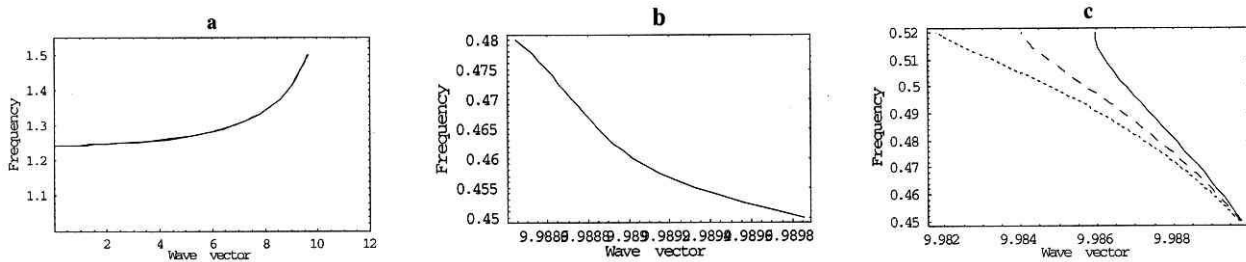


Figure 5. Analysis of dispersion plot at different number of layers.

towards the lower frequency end of the gap and converge towards the upper frequency end of the gap whereas the opposite is true for the curves in the zero-phi gap.

#### 4. Conclusion

The results of these investigations show that the analytic behavior of the dispersion curves for the OTS states in the zero-n and zero-phi gaps seems to be more alike as compared to that of the Bragg gap. The dispersion curves plotted in the Bragg gap, zero-n gap and zero-phi gap are dependent on the angle of incidence but independent on the width of layers. Dispersion curves in the zero-n gap and zero-phi gap are sensitive to the number of layers as compared to the curve in the Bragg gap. These properties of OTS are useful in the applications which are sensitive to the angle of incidence and number of layers.

#### References

- [1] I. A. Shelykh, M.Kaliteevski, A.V. Kavokin, S.Brand. R. A. Abram, J. M. Chamberlin and G. Malpuech, Phys. Stat. Sol. (a) (2007) 204.
- [2] Y. G. Zhu, W. L. Hu, and Y. T. Fang, Opt. Electronics Review **21**, No. 3 (2013) 338.
- [3] I. Shelykh, G. Malpuech, Appl. Phys Lett. **87** (2005) 261105.
- [4] R. Butte and N. Grandejean, Semicond. Sci. Technol. **26** (2010) 014030 (9pp).
- [5] P. Bhattacharya, B. Xiao, A. Das, S. Bhowmick and J. Heo, Phy. Rev. Lett. **110** (2013) 206203.
- [6] C. Schneider, A. Rahimi-Iman, N. Y. Kim, I. G. Savenko, I. A. Shelykh and A. Forchel, Nature **497** (2013) 348.
- [7] M. E. Sasin, R. P. Sesiyan, M.A.Kaliteevski, S. Brand and R.A. Abram, Appl. Phys. Lett. **92** (2008) 251112.
- [8] C. Xue, H. Jiang and H. Chen, Opt. Express **18** (2010) 7479.
- [9] G. Du, H. Jiang, Z. Wang, Y. Yang, Z. Wang, H. Lin, and H. Chen, J. Opt. Soc. Am. B **27** (2010) 1757.
- [10] Z. Chen, C. W. Lenug, Y. Wang, M. Hu and Y. Chen, Opt. Express **19** (2012) 21618.
- [11] K. M. Ho, C.T. Chan and S. M. Soukoulis, Phys. Rev. Lett. **65** (1990) 3152.
- [12] V. S. Veselago, Sov. Phys. Usp. **10** (1968) 509.
- [13] D. R. Smith, W.J. Padilla, D. C. Vier, S.C.Nemat-Nasser and S. Schultz. Phys. Rev. Lett. **84** (2000) 4184.
- [14] R. Rupp, Microw. Opt. Techn. Lett. **38** (2003) 494.
- [15] H. Jiang, H. Chen, H.Q. Li, Y. Zhang, J. Zi and S.Y. Zhu, Phys. Rev. E **69** (2004) 066607.
- [16] P. Markos and C.M. Soukoulis, Wave Propagation from Electrons to Photonic crystals and Left Handed Materials, Princeton University Press (2008).
- [17] R. Burkner, M. Sudzius, I.S. I. Hintschich, H. Fro, V.G. Lyssenko and M.A. Kaliteevski, Appl. Phys. Lett. **100** (2012) 062101.
- [18] I. V. Treshin, V.V. Klimov, P. N. Melentiv and V. I. Balykin, Phys. Rev. A. **88** (2013) 023832.