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SURVEY ON APPLICATION OF LAGRANGIAN RELAXATION, INTERIOR POINT AND DYNAMIC PROGRAMMING FOR THE SOLUTION OF HYDROTHERMAL SCHEDULING

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This paper provides a critical analysis of research work published on the topic Hydro thermal Coordination problem using Lagrangian Relaxation, Interior Point, and Dynamic Programming. The HTS is a large scale, mixed integer, complex, and nonlinear problem. Lagrangian Relaxation fails to find feasible solution. Moreover, it doesn't converge to global minima. Interior Point uses the interior of a given solution instead of using the vertices. Dynamic Programming requires long computation time and storage memory. In addition there is also one more drawback of DP is "curse of dimensionality". The drawbacks of these techniques are resolved by the usage of modern techniques like, co-evolutionary, differential co-evolutionary, stochastic programming, GA, BFA, and PSO etc. At the end a conclusion has been presented based on the literature survey.

Keywords : Lagrangian Relaxation (LR), Interior Point (IP), Dynamic Programming (DP), Hydrothermal Scheduling (HTS)

1. Introduction

Thermal unit commitment, dispatch and hourly generation of hydro units is main concern in hydrothermal scheduling of a power system. The objective function is minimized considering operating cost over a period say a week or a month subjected to constraints i.e. demand and reserve requirements, and individual unit constraints. The mixed-integer problems have been extensively researched for many decades because of potential cost saving. Lagrangian Relaxation method is a technique used to solve constrained optimization problems. Its basic idea is using Lagrange multipliers to relax system-wide demand and reserve requirements [1].

In power system optimization problem the optimal power flow (OPF) is very hot research topic since the last 4, 5 decades [2]. A number of nifty algorithms are applied for HTS: Linear Programming [3], Quadratic Programming [4], Newton's Method [5] are among them. The IP has been applied successfully to the HTS problem [6-9].

DP a conventional heuristic optimization technique tries to reach optimal point upto required accuracy. This technique is useful in solving a variety of problems and can greatly reduce the computational effort in finding optimal trajectories or control policies [10].

Due to the complexity present in the scheduling problem of hydrothermal can be divided into several models and planning horizons, the large scale scheduling problem can be used as input to the medium scale scheduling which in turn provides data for the short term scheduling problem. A large scale scheduling problem is discussed with discrete and continuous variables have to be formulated. Advantages of LR are that: it splits problem into several simple and easily solvable sub-problems as well as lower bound for optimal objective function is provided, but lacks in finding feasible solution for non-convex problems. Lagrangian Relaxation method can be used in combination with other techniques. Lagrangian Relaxation (LR) alongwith augmented Lagrange is used [11].

Multiple constraints are present, so natural approach for HTS problem is decomposition techniques. Separating hydro from thermal generation based on marginal costs by dispatching hydro plants and to complete hydro generation used thermal plants in an iterative algorithm. Problem decomposition in efficient way is to use LR, particularly when unit commitment constraints are involved, because these constraints couple all the generators at every time step. While considering electrical network and line flow limits in

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problem, number of these coupling constraints becomes too large. To overcome this hurdle LR is used in combination with variable splitting (LR-VS) [12].

In power generation by using hydro and thermal units, power generated by hydro thermal units, power generated by hydro units replaces high fuel cost thermal units during peak loads. Thermal units with high fuel cost during peak load are replaced by pumped-storage power plants. Based on their efficiency, fuel cost, responsive velocity, the thermal units are taken separately as base, middle, and peak units. Therefore, it is important how hydro and thermal units are scheduled. However, HTS has been characterized as a large scale, nonlinear, non-convex, and mixed-integer combinational optimization problem. Lagrangian Relaxation updates Lagrangian multipliers based on the degree of system constraints violation. GA searches out optimum using multiple path searching and the ability to handle discrete and continuous variables. But it lacks in finding penalty factor for the hydrothermal scheduling. Fortunately, Lagrangian Relaxation relaxes the constraints using Lagrangian Multipliers. The co-evolutionary algorithm based on Lagrangian is used [13].

2. HTS Problem Formulation

Usually, latest data collection phase in power system ends at 10:00 AM and a feasible solution must be computed by independent system operator as soon as possible. Unambiguous solution is needed for this problem which requires modeling in detail. Based on these requirements, we present a detail model for problem:

The objective function for HTS problem is given by:

Minimize:

$$\mathbf{min\,f} = \sum_{\mathbf{j=1}}^{\mathbf{N}} \mathbf{n_j f_j (PT_j)}$$

Where

$\boldsymbol{j = 1, 2, \ldots, N}$-------Interval of schedule

$\boldsymbol{f_j(PT_j)}$ -------------Thermal generation cost Rs./hr

$\boldsymbol{n_j}$---------------------No. of hrs of jth interval

The objective function above stated is subjected to the following constraints.

A. Load Balance Constraints

$$\boldsymbol{(P_{dj} + P_{loss\,j}) - (PH_j + PT_j) = 0}$$

B. Transmission Line Loss

$$\boldsymbol{P_{loss} = k(PH_j)}$$

C. Hydro-Discharge Constraints

The hydro generation is considered to be a function of discharge rate only.

$$\boldsymbol{q_j = g(PH_j)}$$

Water discharge rate limits are as below

$$\boldsymbol{q_{max} > q_j > q_{min}}$$

D. Thermal Generation Constraints

$$\boldsymbol{PT_{max} > PT_j > PT_{min}}$$

E. Hydro Generation Constraints

$$\boldsymbol{PH_{max} > PH_j > PH_{min}}$$

F. Initial and Final Reservoir Storage Volumes

The initial and final storage volumes are as given below.

$$\boldsymbol{V_H(i,t)|^{t=0} = V_H(i)^{begin}}$$

$$\boldsymbol{V_H(i,t)|^{t=T} = V_H(i)^{end}}$$

3. Other Optimization Techniques used for HTS Problem

Due to the importance of hydrothermal scheduling, this area is extensively researched. A lot of techniques have been applied for HTS problem. We'll discuss briefly one by one. These techniques are discussed briefly as below.

3. Classical Derivative Based Techniques

The classical derivative techniques are based on Newton and Gaussian mathematical techniques. These are very simple and easy to apply. They are usually applied for the simple and small HTS problems having smooth and linear fuel curves but they lack in finding solution for the differential problems i.e. non-smooth curves. These techniques are outdated now. They are limited to theory in these days.

3.1. Deterministic Approaches

The deterministic approaches are the most successful strategies; i.e. inner approximation, outer approximation, cutting plane method, branch and bound methods etc. But here we shall discuss briefly the approaches which are categorized in this branch are; Lagrangian Relaxation, Benders decomposition, mixed-integer programming, dynamic programming, linear programming, and nonlinear programming [14].

3.2. Heuristic Approach

These techniques are nature inspired. They are simulating the behavior of natural phenomena based on knowledge and artificial intelligence. Some of them are evolutionary based i.e. genetic algorithms (GA), differential evolution (DE), particle swarm optimization (PSO), honey bee algorithms (HBA), evolutionary programming (EP), evolution-nary computation (EC), bacterial foraging (BFA), and simulated annealing (SA). The other types are expert systems (ES), artificial neural networks (ANN), and shadowing fuzzy logic (SFL). The important thing about these techniques is that they are mostly adopted by HTS.

We shall briefly discuss them as below.

3.2.1 Genetic Algorithms

The most efficient technique used for the non differentiable optimization problem. It deals with complex, and chaotic search space and multiple constraints. Its search is iterative. We take start by considering a set of solutions randomly selected. They are known as population. By following GA we create next population hoping that the next generation is more reliable, efficient and greater fitness. GA follows the Darwinian Theory (natural selection and survival of fittest). The operators used in GA are generating population, encoding, cross over (reproduction), mutation. All these operators are highly problem dependent and randomly chosen. Although it is very efficient in computation but it is very time consuming and takes a large number of iterations to converge the solution. GA can also be stuck in local optima. They are very efficient for maximization, but little slow for minimization. In HTS we seek to minimize operating cost of the system. GA relaxes constraints using penalty factor but it is not very easy to determine the penalty factor where we use LR [13].

3.2.2. Particle Swarm Optimization

It is popular random based optimization method developed by Kennedy and Eberhart [14] which simulates the behavior of a flock or swarm of birds, and animals. They are initialized with a position and initial velocity in the search space and then they compete for the best position in the search space. It is simple and very uncommon technique but it is very popular in HTS problem. It is used because of its robustness, easy implementation to control parameters, and computational efficiency. In [15] employed different PSO techniques integrated with penalty functions to solve the HTS problem. It requires small number of parameters. It moves to the global optima in iterative manner, also considering the neighboring suitable paths. It has the drawbacks of having very high convergence time. It is mostly used in combination with other techniques as mentioned [16] to handle the inequality constraints.

3.2.3. Simulated Annealing

Another popular algorithm used for complex and chaotic optimization problems. It simulates the behavior of annealing, a process used in thermodynamics. Annealing is a process in which molten metal is cooled slowly; temperature is decreased in such a way to optimize the characteristics, and strength of the metal. The same process is applied in this technique. It is also takes a lot of time for convergence because of large computation requirement of SA it needs to improve it speed [17-18]. If the search doesn’t find the global optima the whole process has to be restarted. It is also used in hybrid with other techniques. Hybridization is done for the sole purpose of to tackle the problem faced by SA and to make overall algorithm efficient. It is similar to GA but differs in operators used. These are used to overcome the shortcomings of conventional gradient based techniques [19]. Evolutionary programming (EP) is very efficient and does not place any restriction on the shape of cost curves and nonlinearities involved [20]. It uses randomly selected solution from search space for initialization and search for the global optima in iterative manner. It has the same pros and cons as those of GA.

3.2.4. Honey Bee Algorithm

It is newest technique introduced which simulates the foraging and evolutionary behavior of honey bees. When comparing with other evolutionary techniques i.e. GA, SA, and PSO it gives very promising results. However, it is in its infantry but it is well explained and defined algorithm. It is well suited for optimization problems like economic dispatch and HTS.

3.2.5. Neural Networks and Fuzzy Logic Techniques

NN mimics the human brain. It memorizes the pattern using past knowledge, and makes guesses based on the previous training. Learning process is through the tuning of the neurons. They are tuned by storing and linking. After the process of learning, this system has the ability to provide the output. It is versatile technique but it is very difficult to design. The number of hidden layers for a problem is very hard to find. Performance is based on the number of hidden layers. It is iterative procedure to find out the optimum number of hidden layers. It is a powerful algorithm but it is itself an optimization problem. Its use for HTS problem is very limited. It was used decade earlier for scheduling [21-22].

3.2.6. Bacterial Foraging Algorithm

Bacterial foraging algorithm is the newest one. It simulates the behavior of E.Coli bacteria, this process was introduced by Kein M. Passino where it is used to optimize distributed controller [23]. Although just like honey bee algorithm it is well defined and well explained algorithm but it lacks in the convergence of very complex, and dynamic problems because it has randomization in nature.

3.2.7. Bender's Decomposition

It works on the simple principles as the Lagrangian Relaxation. The main problem is divided into sub problems and then each problem is solved by relaxing the other sub problems. It is mostly used as hybrid approach to overcome oscillation around global minima, never quite reaching optimum in HTS problem.

4. Interior Point

The Interior Point method is widely used for large scale problems. The HTS problem is complex, nonlinear, non-convex involving constraints both equality and non-equality and bounded as well. The IP method alongwith the Guass-Newton method is presented [24]. For the minimization of residue of Minimum Square of nonlinear equation system is done using Guass-Newton. The linear and nonlinear constrained problem is handled using IP method. The Hessian of nonlinear problem is complex and very expensive in terms of analysis and computation as well. The Hessian terms of nonlinear constraints are depleted for the solution of nonlinear systems. It doesn't affect the convergence of the IP method. The idea for the solution of nonlinear system due to the iterations of IP method is Stationary Guass-Newton method.

The LTHS is very difficult task due to the following reasons: the analysis burden, load demand, stochastic nature of water inflows, dependency of cascaded hydro plants and nature of hydro generation and thermal costs functions are nonlinear. The LTHS is being viewed as an optimization problem with known forecast inflow having each hydro plant represented with its own operational constraints. Mostly the techniques suggested for this problem are network flow based. The IP method as an alternate technique is presented [25]. In this paper the approach proposed is implemented in MATLAB 6.1 and Brazilian Hydropower System having installed capacity of 66,858 MW is used for the evaluation.

Although a number of optimization techniques are applied for the HTS problem and economic dispatch. But they all neglect to consider the network constraints or they require the decomposition of the problem into hydro and thermal part. The IP method utilizes the bordered block structure and Newton system's sparsity and results in a very fast and efficient algorithm. It accounts all type of network traits. In [26] IP method for multi-period hydrothermal economic dispatch is applied considering all the transmission network constraints without requiring the decomposition of main problem.

The IP in combination with heuristic algorithm (GA) is presented [27]. GA is used for the binary variables ON/OFF status of thermal units and the solution of hydraulically coupled of thermal and hydro units is obtained using IP method. The temporal constraints of cascaded reservoir and maximum up down ramps of thermal units are also considered. The flavor of IP Primal-Dual Logarithmic Barrier IP in combination with GA is used.

The convergence time for the solution of the large scale problem HTS is very perilous. A comparison of direct and indirect methods used to solve HTS problem in terms of computation is carried out [28]. In this paper main contribution is based on to have the Quasi-Optimal solution in reasonable time. This is achieved by the usage of direct method in combination with the indirect method approach. The Primal-Dual IP (direct method) relaxes the binary variables of thermal unit's status. The LR (indirect method) is used for the decomposition of the primal problem into hydro and thermal sub-problems. The resulting hydro and thermal units are solved using DP and cutting plane method is used for the maximization of dual function. The result has shown that both the techniques provide the solution practically equal. However, LR is faster.

The centering position inherent to the IP method is very efficient and economical in time consumption. It also provides reasonably accurate solution for classical Optimal Power Flow (OPF). In [29] IPM is used for the hydrothermal optimal power flow (HOPF). The main difference between the OPF and HOPF is that the later is an optimization of dynamic nature. The algorithm has been tested on the six systems, the largest of 1047 buses with 72 time intervals. The following results were observed (i) the algorithm has the capability of handling the large scale problem even that of HOPF and very fast as well compared to other existing techniques (ii) the accuracy of 99% has been achieved and the CPU time is half.

The clipping off method has been merged with IP method [30]. The setting of control variables to their upper and lower bound is the main advantage of the clipping off method. When the certain conditions are met then these variables are depleted from the variables of the main problem. The number of iterations and trials can be reduced as compared to the standard IP solution. No doubt the results are same. It is the first time that IP in combination with clipping off method is used for HTS.

A decoupled method based on Lagrangian Relaxation with KKT (Karush-Kuhn-tucker) conditions of the primal point is proposed for HTS alongwith an efficient algorithm. HOPF is decomposed into OPF sub-problems of thermal and hydro plants [31]. It is suitable for the large scale problems. It greatly reduces the memory requirements. The computation is fast and quick convergence is achieved by using Pi updating method. The reduction in total iterations and execution time is due to the warm started IP quadratic programming. It suits best for practical application because of combined network flow model for variable and fix head plants.

In [9] (IPNP) for OPF on the basis of perturbed KKT conditions of primal problem is proposed. With the help of reduced correction of equations a novel data structure is established. It deals the four objective functions and two types of data structures. Instead of logarithmic barrier function IPNP algorithm is used. No doubt they are mathematically alike. The unification of OPF, C-PF, and A-OPF is done by the utilization of the concept of centering direction. To handle the inequality constraints are handled by derivation of reduced correction algorithm. The use of novel data structure reduced greatly the execution time and fill-ins in comparison to the conventional data structure.

A comparison of different codes of IP applied to the medium term HTS has been carried out [32]. The advantages and disadvantages of commercial and researched codes are conversed. The following codes are studied: CPLEX 3.0 Barrier, HOPDM by Gondzio, LOQO by Vanderbei, PCx by Mehrorta, LIPSOL by Zhang and IPAI. They are all tested on Spanish Hydrothermal System.

5. Dynamic Programming

Implementation of DP in HTS in 1960 was first suggested by B. Bernholtz et al, in which they modeled short term HTS problem of a system having eight hydro and four thermal generators [10].

In [33] The Risk-Constrained Stochastic Dynamic Programming Approach (RCSDP) discuss the Operation Planning of Hydrothermal System. The techniques of RCSDP were applied on the Brazilian generating system which is hydro-dominated and characterized by large, multi-year reservoirs and satisfactory results were obtained.

Another effort was made to reduce long computation time, large storage memory requirements and production cost, the techniques used for this was multi-pass DP combined with successive approximation [34]. The STHTS involves the hour-by-hour scheduling of all generation on a system to achieve minimum production cost for the given time period T.

To reach at the solution first of all feasible regions were calculated by considering only discrete values of time and quantized values of the control and state variables. The upper and lower limits for control and state variables are well defined. Due to Multi-Stage Decision Process unlike the conventional DP, in which the minimum cost-to-target is computed at each stage, at each state a more optimal solution is determined. The reasonable improvement in results, the fast convergence and small memory requirements make the algorithm suitable for practical systems with many generation units.

Same techniques of multi-pass DP with successive approximation was used [35] for short term HTS problem. They used this technique without the requirement of initial feasible solution and also the technique is able to detect the infeasible problems systematically. Case study was performed on the system of Turkish Electricity Authority. They concluded that the approach offer many advantages over the other conventional techniques like non-requirement of initial feasible solution and computation of a spectrum of solutions given by piece-wise linear functions of the system load.

In [36] new techniques of Extended Differential DP and Mixed Coordination were used for HTS problems, which discuss decomposition of the problem into a thermal sub-problem and a hydro sub-problem by relaxing the supply demand constraints. The thermal sub-problem was solved analytically but the hydro sub-problem was further decomposed into a set of smaller problems which were solved in parallel. The advantages were:

1. The dimension problem is avoided.
2. Due cost-effective parallel processing technologies, substantial speedup can be obtained under a parallel processing environment.
3. A quick and accurate estimate on the impact of a change in natural inflow on the total cost.

Test results shown by authors indicated that the algorithm is numerically stable and significantly speedup the programming. Another achievement is that unpredictable changes in natural inflows are effectively handled. For future research work he predicted finding a more efficient nonlinear programming technique for solving hydro-thermal scheduling.

The technique used [34-35] works well to reduce computation time and storage space but it suffer from dimensionality problems also one reservoir is optimized at a time and the operations of other reservoir are kept constant at that proper time so to overcome these problems a new technique of DP two-stage algorithm is used [37]. In this algorithm all allowable states in reservoirs are restricted to those stages before and after the supposed stage. Here Discretization of state and control variables is not required, so dimensionality issue is solved. The proposed method is used to minimize the sum of operation costs in two consecutive periods, providing that the HTS problem as a function of reservoirs' water content in one period. The algorithm has reduced storage space requirement and also have less computing time requirements as compared to that of DP with successive approximations method. As Discretization of state and control variables is required in successive approximations method so it ignores the dependence of the operation policy of one reservoir on the actual water content of other reservoirs. In order to reach at the optimal operation policy for one reservoir is assumed constant and difficulty in considering complex hydro networks characteristics is created. Using DP two-stage algorithm approach no Discretization of state and control variables is needed neither transformation in equivalent hydro chains is required. Hence the detailed modeling of the hydro network is possible.

A comparative study is then performed between Lagrangian Relaxation and truncated dynamic programming methods [38]. The commitment states of the system are obtained by solving thermal units and thermal sub-problems using truncated DP and LR respectively. Load demand, spinning reserve, the capacity limits, minimum up and down time, the ramp rate and the hydro constraints are also considered in the problem formulation of both methods. Non-linear cost function and dispatches were used and accurate transmission losses were incorporated. The two methods are compared for speed of execution and operating cost by testing them on a practical utility system. As the computation time of DP is very high but the truncated version reduces real computing requirement with a possible loss of accuracy. LR is the least precise of the mathematical programming techniques but it provides the best performance in computing complex problems. In solution of HTS first the commitment states of thermal units is calculated and then an efficient hydrothermal scheduling algorithm was developed to solve the output levels of hydro units. Due to relaxation of constraints a gap which does not satisfy power balance and reserve constraints, to overcome this problem the Lagrangian Multiplier are adjusted using linear interpolation. Using TDP various coupling constraints can be handled more easily. In truncated version, a less optimal solution is obtained due to fixed priority ordering for commitment among units. For large scale units, its performance is not as satisfactory as truncation is used. The LR method has more flexibility as no priority ordering is imposed. For large systems, it is computationally much more efficient. Its main flaw is that the dual optimal solution does not usually satisfy the once relaxed coupling constraints. Due to Lagrangian multiplier and TDP the commitment state may be disturb so to overcome this problem a refinement algorithm is applied.

Another comparison study is performed between Primal and Dual Stochastic DP [39] with the aim to remove curse of dimensionality problems. The comparison was made by simulating the historical inflows records of Brazilian Hydroelectric system, by taking only one hydro plant. Stochastic variable of the system was modeled by a lag-one parametric auto-regressive model. In dual approach, a parametric auto-regressive model of superior model is also considered. Results were showing that the performance of the primal and dual stochastic DP is different. The worth of the DP is decreased due to the curse of dimensionality problem, as the computational burden increases exponentially with the number of state variables. Various approaches have been recommended to overcome this problem including the aggregation of the hydroelectric system through a composite representation and the use of dual stochastic DP, based on Bender's decomposition. DSDP avoid the Discretization of the state space in the solution of the recursive equation of the DP. The approach uses the piecewise linear functions to approximate the expected cost-to-go function of SDP at each stage. These approximate functions are achieved from the dual solution of the problem at each stage, due to the Bender's decomposition.

To identify the effect of different stream flows on the stochastic DP a study is performed on SDP for Long term HTS problem considering Different Stream Flow models in [40]. By considering different stream flow models progressively complex one, the benefits of increasing sophistication of stream flow modeling on the performance of stochastic dynamic programming was identified. The first simplest model taking the inflows by their average values; in the second model inflows were taken as independent probability distribution functions; and the third model adopts a Markov chain based on a lag-one periodical auto-regressive model. The effects of using different probability distribution function have been also addressed. Numerical results for a hydrothermal test system composed by a single hydro plant have been obtained by simulation with Brazilian inflow records. From the simulation results it was concluded that both the deterministic and stochastic approaches have provided quite similar performance in both case studies. Differences are quite small in terms of hydro generation and cost which means that different stochastic models considered do not provide significant improvement in dynamic programming for long term hydrothermal scheduling. The results shows that deterministic approaches can easily handle multi reservoir systems without the need of any modeling manipulation and therefore, has significantly less computational efforts as compared to stochastic approaches.

In order to reduce computation time while considering different spinning reserve requirements a research study was performed [41] and the techniques used in this study are the combination of Hybrid EP and Multi-Pass DP. Two types of spinning reserve requirements are considered in this paper, these are frequency relating reserve requirements and instantaneous reserve requirements. FRRR is used to control frequency of the system and IRR is used to replace a nonfunctional unit quickly. So hydro and combustion units are used for IRR and hydro units are used for IRR and hydro units with automatic generation control is suitable for FRRR due to quick response. First a two stage method was used for scheduling of multi-reservoir system, the computation time of the method was fast and without Discretization of state, but it did not consider the characteristic of individual thermal unit and FRRR and IRR of system were also not involved. So to overcome this problem multi pass dynamic programming is used which is fast and requires less storage memory but it takes solution from three discrete values. For better performance Evolutionary programming is combined into MPDP.

The main function of hydro unit described was to shave the peak load in conventional hydrothermal scheduling mode. Whereas, comparison of the frequency response rates showed that hydro units had an excellent frequency response rates. The results of simulation showed outstanding results in the favor of algorithm but here curse of dimensionality occurs.

A detailed case study was then performed [42] which is on Sampling Strategies and stopping criteria for stochastic dual DP. The problem was formulated to obtain an optimal policy, under water energy resources uncertainty, for hydro and thermal plants over multi-annual planning horizon. The problem was modeled as a multistage stochastic program and an algorithm was developed for it. The original stochastic process shows a finite scenario tree and, because of the large number of stages, a sampling based method such as the Stochastic Dual Dynamic Programming algorithm is required. The purpose of this paper is two-fold. Firstly, the application of two alternative sampling strategies to the standard Monte Carlo namely, Latin hypercube sampling and randomized Quasi-Monte Carlo for the generation of scenario trees, as well as for the sampling of scenarios that is part of the SDDP algorithm was studied, and secondly the formulation of stopping criteria for the optimization algorithm in terms of statistical hypothesis tests was discussed, which allows to propose an alternative criterion that is more robust than that originally proposed for the SDDP. These ideas were tested on a problem associated with the whole Brazilian power system, with a three year planning horizon. The quality of the solution produced by the algorithm by means of a test of the radio of upper and lower bounds that considers a tolerance for the estimation error was then proposed. The suggestion for the technique is to apply LHS methods for the generation of scenarios from a given tree. RQMC methods can also be used for that purpose as well if the number of stages is small or an RQMC method that can handle well relatively high dimensional vector is used. At the end the authors recommended that RQMC appears to be useful for the future work in the field of generating the scenario trees if only low-dimensional vectors are required as it is in the case of hydrothermal coordination for long term planning.

6. Lagrangian Relaxation Method

In the field of mathematical optimization, Lagrangian Relaxation is a relaxing method which approximates a difficult problem of constrained optimization by a simpler problem. By using Lagrangian Relaxation violation of inequality constraints is penalized, posing a cost on violations. The problem of maximizing the Lagrangian function of the dual variables is the Lagrangian dual problem.

Although, a variety of techniques which are advance and computationally efficient than LR exist these days but still we need Lagrangian Relaxation to satisfy the constraints. Lagrangian Relaxation is an excellent and best suited method to satisfy the constraints of the problem present days. The co-evolutionary techniques are used to find optimum values in HTS problem. But they use the solution provided by the LR to optimize the objective function. It is present in these days and other advance and nature inspired techniques in particular are used to overcome the difficulties faced by the system.

Lagrangian Relaxation is used in combination with the Augmented Lagrangian. The usage of Augmented Lagrangian is to cope with the inability of Lagrangian Relaxation to find a near feasible solution for nonlinear, nonconvex, and complex optimization problem [1]. There some invariants of method are also used because problem is not separable. The usage of decomposition technique made Lagrangian Relaxation very efficient. The Lagrangian Relaxation in combination with artificial variables technique is presented [11]. It has made the introduction of new constraint variables possible. Two phase approach is very efficient if we consider hydro production is modeled using nonlinear programming and hydro mixed integer constraints are considered. Lagrangian Relaxation relaxes the constraints but hydro sub problems are still coupled in space and time. Lagrangian Relaxation in combination with variable splitting (LRVS) used for thermal and hydro variable duplication as well as turbine outflow and spillage variables [12]. As LRVS cannot find the feasible solution, AL is used to tackle this issue. A robust feasible solution is obtained, future cost function is considered is clearly continuous nonlinear, 1-0 binary variables for ON/OFF states and forbidden operating zones are also considered. The problem considered is large scale involves nonlinearity. When electrical network flow and line flow limits are considered in HTS problem number of constraints are too large that they cannot be solved by classic way i.e. applying LR when only thermal units commitment and reserve constraints are considered. This difficulty is overcome by combining Lagrangian Relaxation with Variable splitting. Decomposition is achieved by duplicating some of the variables. The variants bundle method used to solve the dual problem. By using these methods the difference between artificial and original variables is very small while the load attainment was almost satisfied. The infeasibility of solution obtained by the primal solution exist which can be reduced by piecewise linear approximation for the non linear constraints and addition of additional variables to reduce inherent oscillatory effects in primal variables to LR techniques.

The co-evolutionary techniques are also used in combination with the Lagrangian Relaxation method [13]. The method has two steps; Lagrangian Relaxation is used to form the Lagrangian function from primal solution, known as dual function. The co-evolutionary algorithm employs the two genetic algorithms in parallel for the evolution of control variables as1st population and 2nd population for Lagrangian multipliers. The fitness function minimization is used to update the control variables by using 1st population while maximization of fitness function used for the adjustments of multipliers as 2nd population. Control variables and Lagrangian multipliers are updated simultaneously with the help of co-evolutionary algorithms for the performance improvements. The results revealed that proposed method has effectively found the optimal solution.

Due to the presence of oscillation in dual solution inherent to Lagrangian Relaxation, the convergence is not satisfactorily. The nonconvex problem is due to the presence of the integer variables and network constraints. The elimination of violated constraints cannot be done iteratively [43]. Lagrangian Relaxation in combination with piecewise linear approximation of penalty is used to avoid oscillation as well as for the improvement of dual solution. To make Lagrangian decomposable C. Liu at el. used Block Descent Coordination technique to cope with it.

Lagrangian heuristic is used to exploit the information obtained by dual problem solution using primal Bundle Method and “Warm Starting” method that improves both convergence and quality of the solution [44]. Main advantage of this approach is the adoption of the disaggregated methods and exploitation of the primal information available.

The use of Lagrangian Relaxation in combination with the Augmented Lagrangian (AL) while considering transmission constraints is presented in [45]. Transmission constraints are not considered mostly in HTS problem because it increases the complexity of the problem by introducing new constraint variables to satisfy. To achieve feasible schedule extra units can be committed but the deviation from optimality would occur in schedule.

The resulting sub-problems by using Lagrangian Relaxation method are linear or piecewise linear functions. The solutions of these problems oscillate between optima. An augmented Lagrangian decomposition and coordination technique is used [46]. The oscillations and smoother dual function is resulted by appending the quadratic penalty term to Lagrangian. AL greatly reduces the oscillations, increases the speed of convergence and also computationally efficient but it damages the lower bound property.

The power system with cascaded and head-dependent reservoirs is scheduled by presenting a new method to Lagrangian Relaxation method [47]. Due to constraints, hydro river catchments, discontinuous operating regions, discrete operating states and hydraulic coupling of cascaded reservoirs. It is not easy to handle reservoir limits and other constraints coupled in time and space.

Unit commitment of hydro is the key feature in HTS. A realistic approach for the development and implementation for the hydro unit commitment determination is presented [48]. The spatial and temporal coupling relaxation is presented. The Bundle method is used in an algorithm is used to update the Lagrangian multipliers. Sequential Quadratic Programming, Linear Programming, Mixed-Integer Linear Programming, and Bundle Method are used in hybrid with LR to have the solution of the optimization problem.

The comparison of Lagrangian relaxation and Interior Point in terms of their performance for HTS problem is done [49]. Both gave the same results but the Lagrangian shows very efficient convergence. It is speedy and provides faster convergence.

The scheduling of large scale hydrothermal power system is presented [50] based on Lagrangian Relaxation technique. The general problem of that type includes in addition to other factors the following as well: (i) random load demand; (ii) non-linear cost function of thermal; (iii) variation of water head; (iv) nonlinear function of hydroelectric output; (v) regulation of reservoirs with limited spillage capacity in cascaded case. Due the real system consideration such as fluctuation of power interchange cost it gives a very flexible model.

LR solves the dual problem despite of the original problem. The duality gap is as low as 0.3%. The duality gap is defined as the difference between the optimal values of objective function for primal and dual problem. [51] the updating of Lagrangian multipliers is focused. A novel, non-oscillating, and computationally efficient procedure is presented.

The optimal distance method is used to update the multipliers [52]. The method is based on Kuhn Tucker optimality principles. The minimization of the optimal distance function results in the satisfaction of all the constraints. The reduction of optimal distance function means a near optimal and feasible solution is obtained. The performance comparison gives better results in term of accuracy and convergence when compared to subgradient method.

In [53] a nonlinear approximation method is introduced to cope with the inherent oscillation of LR. These oscillations cause the large difference in solution of individual subproblems with the solution of primal problem. in this paper the nonlinear approximation utilizes the nonlinear function to solve these subproblems e.g. quadratic function. By using this method the difficulty of solution due to the presence of oscillation is eliminated and the singularity is avoided as well.

Among a number of techniques to solve SHTS problems the mostly used are linear and nonlinear programming. The objective function, Bender's decomposition, and Lagrangian relaxation are most renounced techniques. For the large scale systems the Lagrangian Relaxation method provides the very promising results as compared to others [54]. To formulate the subproblems of the original problem is found to be the most competent and comprehensive approach. In this paper the author introduced the Lagrangian Relaxation method while solving the dual problem.

An improved Lagrangian Relaxation method is presented [55]. The system demand and reserve requirements are satisfied using Lagrangian Relaxation method. The main problem is decomposed into subproblems. The dynamic programming without discretizing generation level is used for thermal subproblem. Extensive constraints e.g. spinning reserve, power balance, minimum up/down time, ramp rate, capacity limits and hydro constraints, transmission losses non linear cost functions are considered. The new method provided better results than the standard Lagrangian Relaxation method.

7. Conclusions

A detailed survey has been carried out on the title. The three techniques used for HTS problem used earlier are discussed and reviewed. No wonder there are other very efficient and computationally fast techniques which, can provide very promising optimal solution in short time. But to satisfy the inherent constraints in HTS problem coupled in time and space Lagrangian Relaxation technique is best suited. The nature inspired techniques like GA, SA, PSO, BFA etc. exploit the results of dual problem solution to achieve their goal. The dual problem is solved using Lagrangian Relaxation method. It must be used in hybrid with these techniques.

The Interior Point method is very efficient and economical in terms of execution time. It is well suited to the large scale problems. The plus point of IP is that it can handle the whole problem. It represents the HTS problem in more detail and doesn't deduct the details of system for the sake of feasible solution. It provides very promising and accurate results when the transmission network constraints are considered. It is not very common in these days due to advance techniques stated above.

Different flavors of Dynamic Programming are discussed in this paper also. First the Incremental Dynamic Programming is used to solve HTS problem but it is time consuming and have curse of dimensionality. Then Multi-Pass DP is used which is fast and use less storage space but it has problem of dimensionality so to overcome this problem extended Differential DP and mixed coordination techniques are used. Then iterative two stage methodology DP techniques are used for long term HTS problem which do not require Discretization and also detail model of hydro plant is possible using this technique. The comparison of LR and truncated DP methods for solving long range HTS problem is discussed and the result shows that LR approach produces lower cost schedules than TDP approach. Comparison between primal and dual stochastic DP in the HTS problem is then performed, which overcomes curse of dimensionality and also DSDP does not require Discretization of state space. SDP approach is used to solve long term HTS considering different stream flows but it has a very little effect on the result, then to improve performance of the system using Multi-Pass DP, evolutionary programming is combined into MPDP.

However, in these days these techniques are little outdated but they are used in combination with the modern techniques to achieve the better results and satisfy the inherent constraints of HTS problem which, cannot be handled by modern techniques alone.

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